PRELIMINARY EXAMS
Mathematical Sciences, UD

FIRST TIER

Linear algebra

Topics covered on the exam include the following. Most of these topics are discussed in Math 672.

1. Subspaces, bases and dimension (Chapters 1 - 2, [1]).
2. Linear transformations and matrix representations (Chapter 3, [1]; Chapters III - IV, [3]).
3. Determinants and rank (Chapter VI, Section V.3, [3]).
4. Inner products and inner product spaces (Chapter 6, [1]).
5. Linear functionals, adjoints, and dual spaces (Chapter 6, [1]; Chapter I. F, [2]).
6. Bilinear forms, Hermitian forms, and Quadratic forms (Sections IV.A - IV.C, [2], Chapter V [3]).
7. Eigenvalues, eigenvectors, and characteristic polynomials (Sections VIII.1 - VIII.2, [3]).
8. Cayley-Hamilton Theorem (Sections III.A - III.C, [2]).
9. Operators on inner product spaces and Spectral Theorems (Chapter 7, [1]; Section III.D, [2]; Section VIII.3 - VIII.6, [3]).
10. Jordan Canonical Form (Chapter XI, [3]; Section III.E, [2]; Chapter 8, [1])

Chapter and section numbers refer to

[1] S. Axler, *Linear Algebra Done Right*, Second Edition, Springer-Verlag, 1997.
[2] M.L. Curtis, *Abstract Linear Algebra*, Springer-Verlag, 1990.
[3] S. Lang, *Linear Algebra*, Third Edition, Springer-Verlag, 1987.

Real analysis

Topics covered on the exam include the following. Except the part on multivariable calculus, most of these topics are discussed in Math 600.

1. Metric Spaces: open and closed sets, compactness, connected sets, complete sets, continuous functions on metric spaces ([1], Chapters 3 and 4, [3] Chapter 2).
2. Continuity and Differentiation: mean value theorem, Rolle's theorem, Taylor's formula, derivatives of vector valued functions, uniform continuity, monotonic functions ([1], Chapters 5 and 6, [3] Chapter 4).

3. Infinite Sequences and Series: Limit superior and limit inferior, monotonic sequences, alternating series, absolute and conditional convergence, power series, tests for convergence of series, rearrangement of series ([1], Chapter 8, [3] Chapter 3).
4. Sequences of Functions: Pointwise convergence, uniform convergence, uniform convergence and continuity, differentiability and integration ([1], Chapter 9, [3] Chapter 7).
5. Riemann integration ([1] Chapter 7, [3] Chapter 6).
6. Functions of Several Variables: Directional derivatives, the total derivative, Jacobians, inverse function theorem, implicit function theorem, extrema problems ([1], Chapters 12 and 13).
7. Vector Calculus: Line integrals, Green's theorem, surface integrals, Stokes theorem, the divergence theorem ([2], Chapters 10, 11 and 12).

Chapter numbers refer to

[1] T. Apostol, *Mathematical Analysis*, 2nd edition, Addison Wesley, 1974.
[2] T. Apostol, *Calculus*, Vol. 2, 2nd edition, John Wiley, 1969.
[3] W. Rudin, Principles of Mathematical Analysis, 3rd edition, McGraw Hill, 1976.

SECOND TIER

Integration and complex variables

Topics covered on the exam include the following. Most of these topics are discussed in Math 602.

1. Construction and properties of the Lebesgue measure
2. Lebesgue measurable and integrable functions
3. The dominated convergence theorem, Fatou's lemma, the monotone convergence theorem, and the bounded convergence theorem
4. Analytic functions, Taylor series, Cauchy's theorem, the generalized Cauchy integral formula
5. The maximum modulus principle and Liouville's theorem
6. Laurent series, the residue theorem and applications to computation of integrals

References are:

- [1], Chapter 10 and [2] Chapters 2-4 and 7 for items 1-3.
- [1] Chapter 16 and [3] Chapters 2, 4, and 5 for items 4-6.

[1] Tom Apostol, *Mathematical Analysis*, 2nd edition, Addison Wesley, 1974.
[2] H.L. Royden, P.M. Fitzpatrick. *Real Analysis*, 4th edition. Person, 2010.
[3] L. Ahlfors, *Complex analysis*, 3rd edition. McGraw-Hill, 1979

Applied mathematics

Topics covered on the exam include the following. Most of these topics are discussed in Math 617.

1. Ordinary Differential Equations
    a. Fourier series (use in PDEs) [1] ch. 3.
    b. Sturm-Liouville theory (as preparation for eigenfunction expansions of PDEs) [1] ch. 5.
    c. Green's functions [2] §5.5
    d. Bessel and Legendre functions (as eigenfunctions for PDEs in alternate geometries) [1] ch. 7.
2. Second-Order Linear PDEs
    a. Separation of variables [1], ch. 2
    b. Fourier and Laplace transform methods [1] ch. 10 and 13
    c. The diffusion equation (maximum principle) [1] ch. 2
    d. Laplace's equation (maximum principle, Poisson's integral formula): [1] §9.5
    e. The wave equation (characteristics, d'Alembert's solution): [1] §12.3
3. Variational principles: [2] §4.1, 4.3

Section and chapter references are given from

[1] Haberman, Richard. *Applied Partial Differential Equations with Fourier Series and Boundary Value Problems,* 5th ed. New York: Pearson, 2013.
[2] Logan, J. David. *Applied Mathematics*, 4th ed. New York: Wiley, 2013.

Numerical methods

Topics covered on the exam include the following. Most of these topics are discussed in Math 611.

1. Polynomial interpolation:
    a. Lagrange and Newton form [QSS, 8.1,8.2].
    b. Piecewise interpolation in 1 and 2D [QSS, 8.3,8.5].
    c. Splines [8.6,8.7].
2. Numerical Integration and Finite Differences:
    a. Newton-Cotes and composite formulae [9.2, 9.3, 9.4].
    b. Singular integrals [9.8].
    c. Orthogonal polynomials [10.1].
    d. Gaussian quadrature [10.2, 10.4].
    e. Approximation of derivatives [10.10]
3. Numerical solution of Ordinary Differential Equations:
    a. Gronwall Lemma [11.1],
    b. One step methods [11.2],
    c. Stability and consistency [11.3]
    d. Difference equations [11.4]
    e. Multistep methods [11.5]
    f. Consistency and stability [11.6]
    g. Runge Kutta methods [11.8]
    h. Stiff Problems [11.10]
4. Finite Differences
    a. Finite difference approximation to two point boundary value problems [QSS12.2]
    b. Discretization of the heat equation [13.2]

The section numbers refer to *Numerical Mathematics* by A. Quarteroni, R. Sacco, and F. Saleri.

Algebra

Topics covered on the exam include the following. Most of these topics are discussed in Math 650.

1. Elementary Group Theory
    a. basics (group, subgroup, cosets, Theorem of Lagrange) (Ch. 1,2)
    b. Homomorphisms, normal subgroups and Isomorphism Theorems. (Ch. 3)
    c. Classification of Finite Abelian Groups. (Ch. 7AB)
2. Commutative Rings
    a. Polynomial rings, PIDs and UFDs. (Ch. 16)
    b. Fields and field extensions, splitting fields. (Ch. 17)
3. Finite Fields (Ch. 21AB)
    a. Structure and uniqueness. Subfields. (Ch. 21A)
    b. Irreducible polynomials (and counting them). (Ch. 21B)

Chapter numbers refer to I.M. Isaacs, *Algebra, A graduate course*.

Stochastic processes

Topics covered on the exam include the following. Most of these topics are discussed in Math 631.

1. Markov chains (discrete-time) (Chapter 1)
2. Continuous-time Markov chain (Chapter 4)
3. Martingales (discrete-time) (Chapter 5)
4. Key examples:
    a. Random walks
    b. Birth and death processes
    c. Branching processes
    d. Markov chain Monte Carlo

Chapter numbers refer to R. Durrett, *Essentials of Stochastic Processes*

Last changed by RJ Braun; Dec 17, 2018.