Mathematical Modeling of Glob-Driven Tear Film Breakup

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Introduction

Tear film thinning is usually found related to thin lipid layer due to a higher evaporation[1]. We define tear film breakup (TBU) as tear film thins less than 0.25μm.

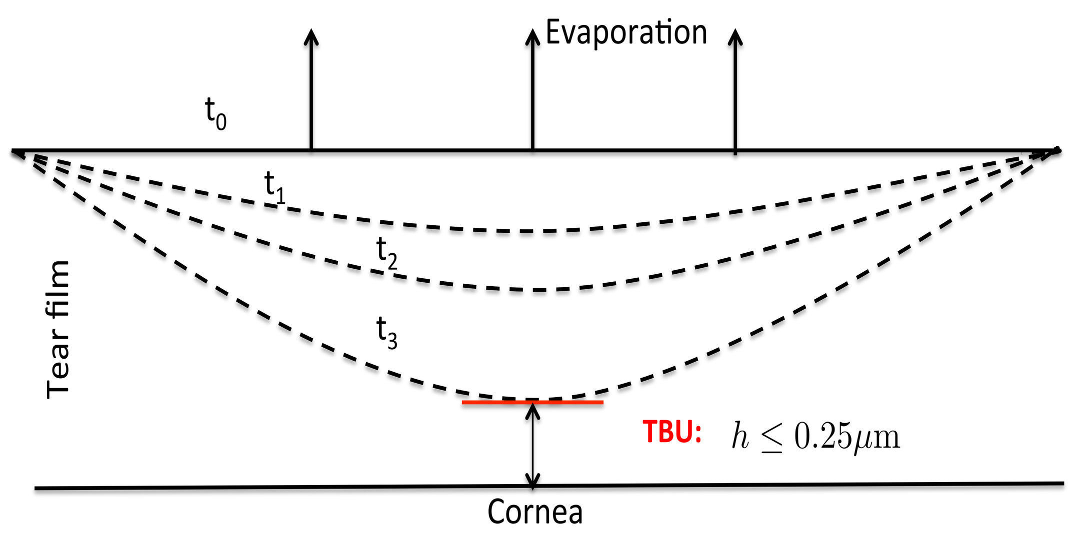
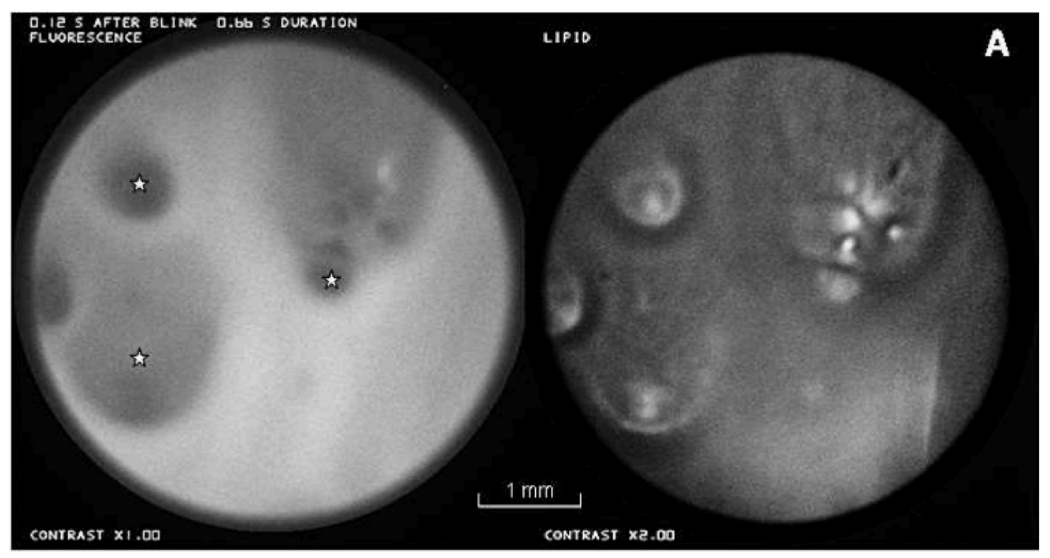


Figure 1: *Time sequences of tear film thinning.*

However, this pattern is not always true. Occasionally, the thinning is observed in some corresponding thicker lipid region (we call it glob), where the evaporation is supposed to be low. We've hypothesize the existence of strong outwards flow under the glob.



- Globs marked by stars.
- Brighter region represent thicker lipid layer in TFLL (right).
- Dark regions represent thinner film in fluorescence picture (left).

Figure 2: *Image of tear film fluorescence (left) compared with image of the tear film lipid layer (TFLL) (right)*[1]

Objectives

We build a mathematical model trying to understand why this will happen.

1. How can we incorporate the outwards flow?
2. Will the outwards flow be strong enough to lead to TBU?
3. How will the physical features of the glob affect the time to TBU (TBUT)?

References

[1] King-Smith et al. IOV Sci. 54.7 (2013): 4900-4909.

[2] R. J. Braun *"Dynamics of the Tear Film"*, Annu.Rev. Fluid Mech. 2012

Outwards flow

We think the glob has a different composition with mobile lipid layer, which has lower surface tension. The ourwards flow is the flow from lower to higher surface tension. The flow is called the Marangoni effect.

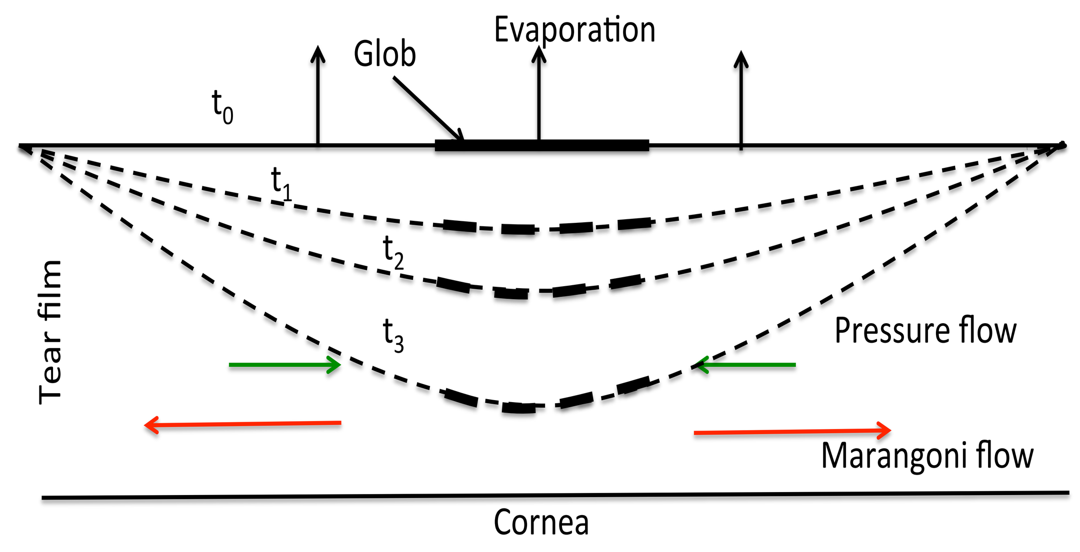
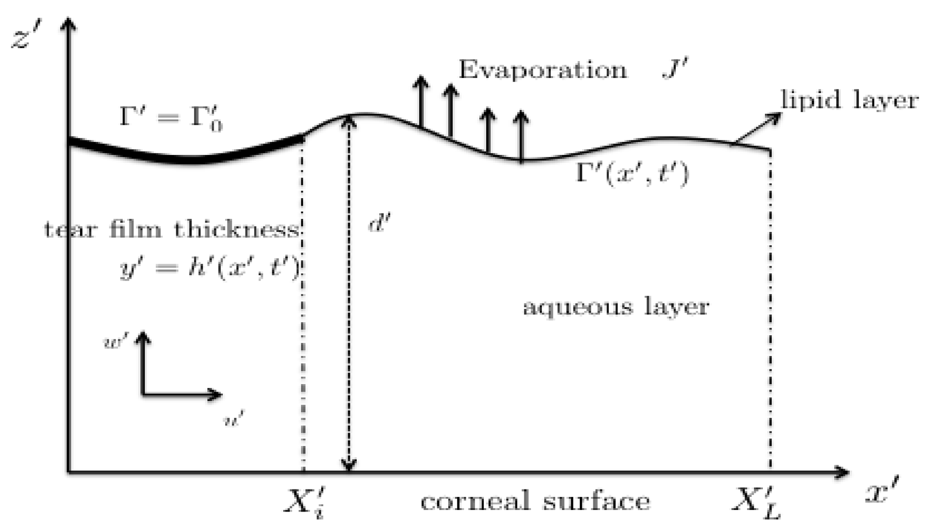


Figure 3: *Idealized time sequences of tear film thinning and components to TBU with glob*

Mathematical Model

We build an axisymmetric spot model based on the following assumptions:

- The glob is symmetric and flexible.
- The glob has fixed higher surfactant concentration thus lower surface tension.



- Tear film thickness (d): 3.5 - 5 μ m
- Domian size $(X'_{L'})$: 0.5 - 4 mm
- $\varepsilon = d/X'_{L'} \approx 10^{-2}$

Figure 4: *Axisymmetric glob-driven model*

Since $\varepsilon \ll 1$, we can use lubrication theory, which helps us simplify our model into two PDEs[2].

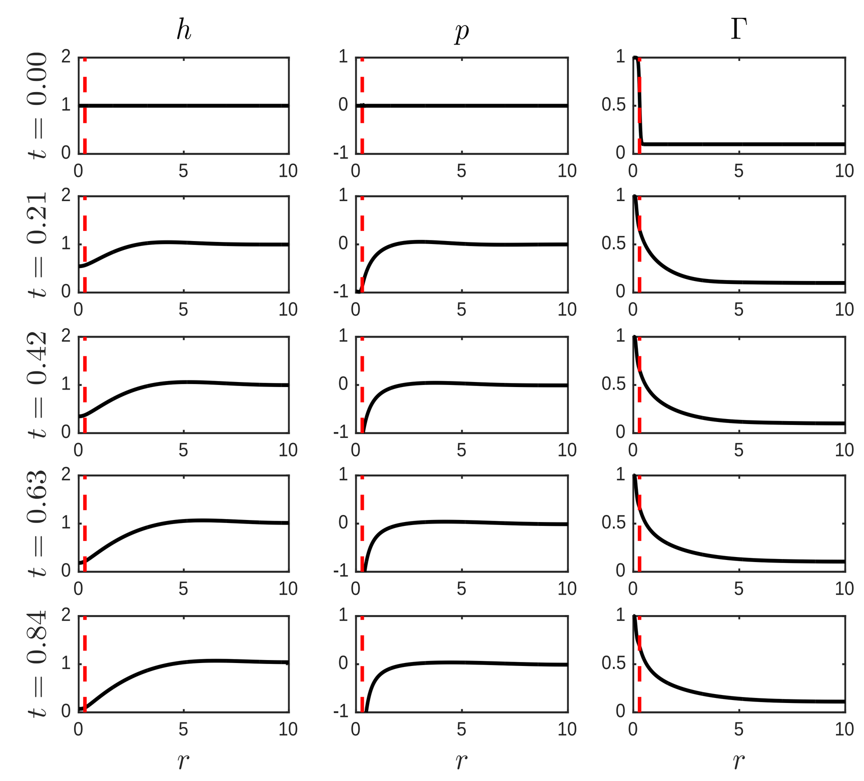
$$\partial_t h + \frac{1}{r}\partial_r(rh\bar{u}_r) = -J(r,t) \qquad (1)$$

$$\partial_t \Gamma = \left[\mathrm{Pe}_s^{-1}\left(\frac{1}{r}\partial_r(r\partial_r\Gamma)\right) - \frac{1}{r}\partial_r(r\Gamma u_r)\right] B(r) \qquad (2)$$

Where $\bar{u}_r$ denotes the average flow and u_r is the surface flow.

$$\bar{u}_r = \frac{-\frac{1}{3}\partial_r p h^2[B + (1-B)\frac{1}{4}h] - \frac{1}{2}Bh\partial_r\Gamma}{B + (1-B)h}, \quad u_r(r,h,t) = \frac{1}{2}\partial_r p h^2 + \widetilde{\mathscr{M}}(B)h \qquad (3)$$

Numerical Results



- Tear film thins at the center of the glob. Strong outwards flow is observed.
- The surfactant keeps spreading out to the outer region of the lipid layer.
- The pressure is trying to push the water back.

Figure 5: *TBU observed at $t = 0.84s$, glob with radius 0.04mm, no evaporation included. Red dashed line represents the edge of glob.*

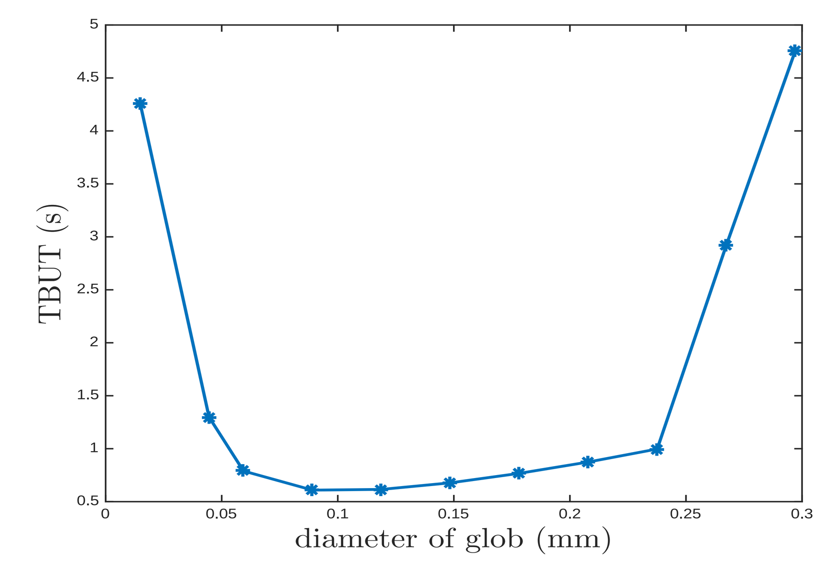


Figure 6: *TBUT and glob size when $d = 3.5\mu m$, no evaporation included.*

Conclusions

1. The Marangoni effect can drive a strong outwards flow which leads to TBU in some cases.
2. Increasing glob size will increase TBUT (down to diameter 0.1mm).
3. Stronger Marangoni effect decreases TBUT (not shown).

Future Work

Add equations for important aqueous solutes, such as osmolarity and fluorescein into the model.

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