Inverse acoustic scattering with reduced data

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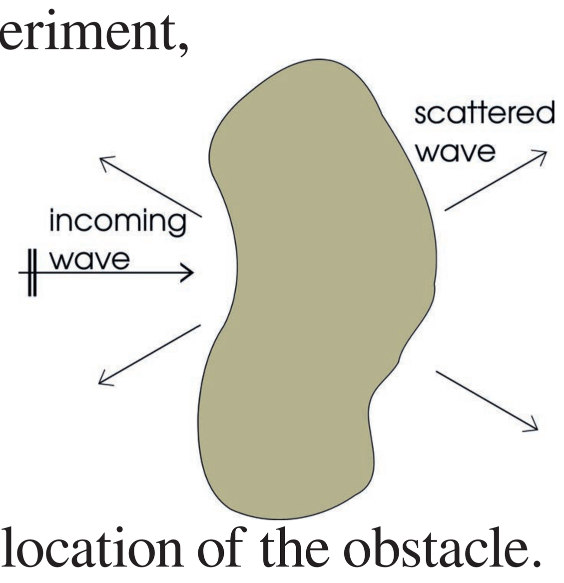
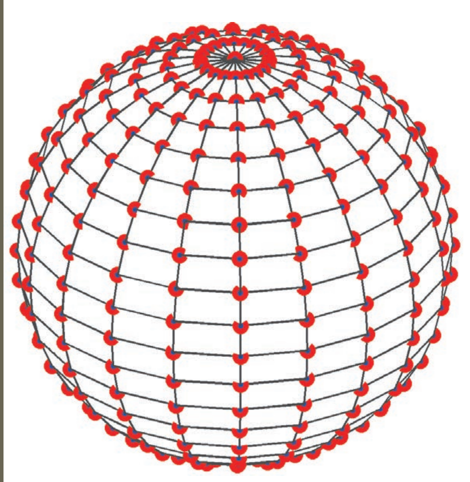
Acoustic scattering experiments

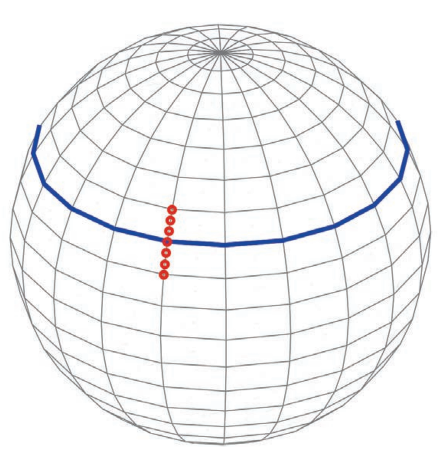
In a scattering experiment, an incident plane wave is directed at an object. From the *far field pattern* of the scattered wave collected by far away receivers, we reconstruct the location of the obstacle.



We are able to find the location of small obstacles using less data than is typically required.



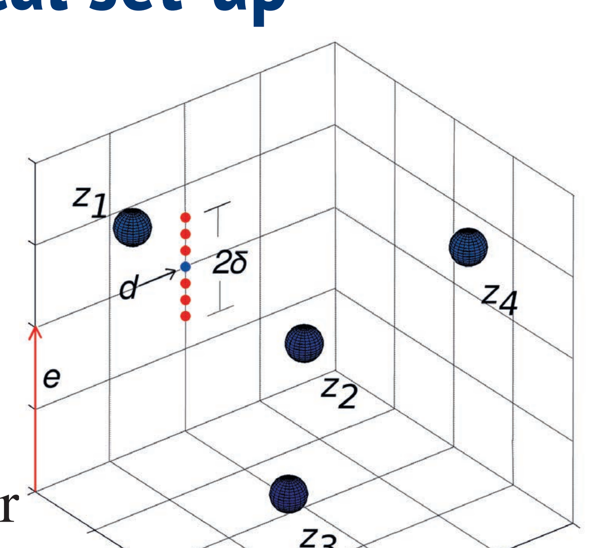
In the typical multistatic set-up, scattered data must be collected by many receivers surrounding an object.



The newly developed quasi-backscattering set-up uses a line of receivers extending a small distance in one dimension from a transmitter.

Mathematical set-up

Assume there are M obstacles at points z_j. Say the obstacles have strength t_j.



For a transmitter located at point **d**, fix a vector **e** which is orthogonal to **d**. Place receiving devices at the locations **x**=-**d**+η**e** where η ranges between the small parameter -δ and δ. The incident field has wave number k.

For small-enough obstacles, the data collected by receivers is

$$u_\infty(\mathbf{d},\eta) = \sum_{j=1}^{M} t_j e^{2ik\mathbf{d}\cdot z_j} e^{-ik\eta\mathbf{e}\cdot z_j}$$

A range test and an inversion scheme

We take an approach for inverse scattering problems and extract information from the far field operator,

$$(Fg)(\mathbf{d}) = \int_{-\delta}^{\delta} u_\infty(\mathbf{d},\eta)g(\eta)\,\mathrm{d}\eta = \sum_{j=1}^{M} t_j e^{2ik\mathbf{d}\cdot z_j} \int_{-\delta}^{\delta} e^{-ik\eta\mathbf{e}\cdot z_j} g(\eta)\,\mathrm{d}\eta.$$

Range Test: if z_j do not lie on the same line parallel to **e** and z is orthogonal to **e**, then

$$e^{2ik\mathbf{d}\cdot z} \in \mathrm{Range}(F)$$

if and only if z is the projection of some z_j onto the plane perpendicular to **e**.

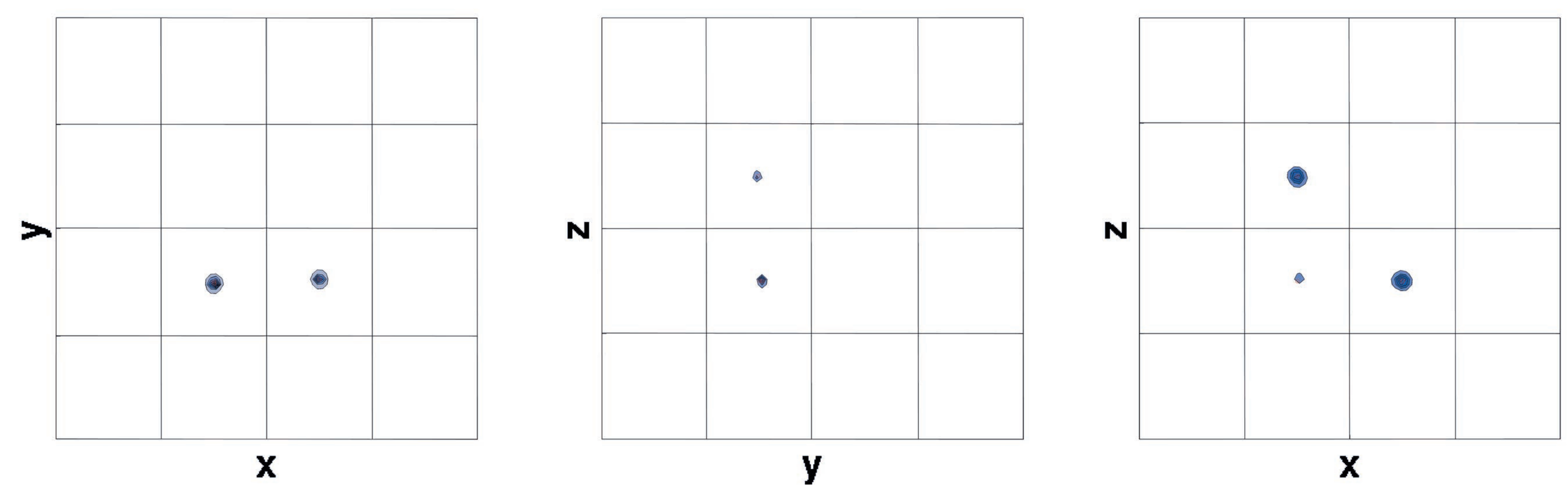
Equivalently, if P is a projection operator onto the orthogonal complement of the range of F,

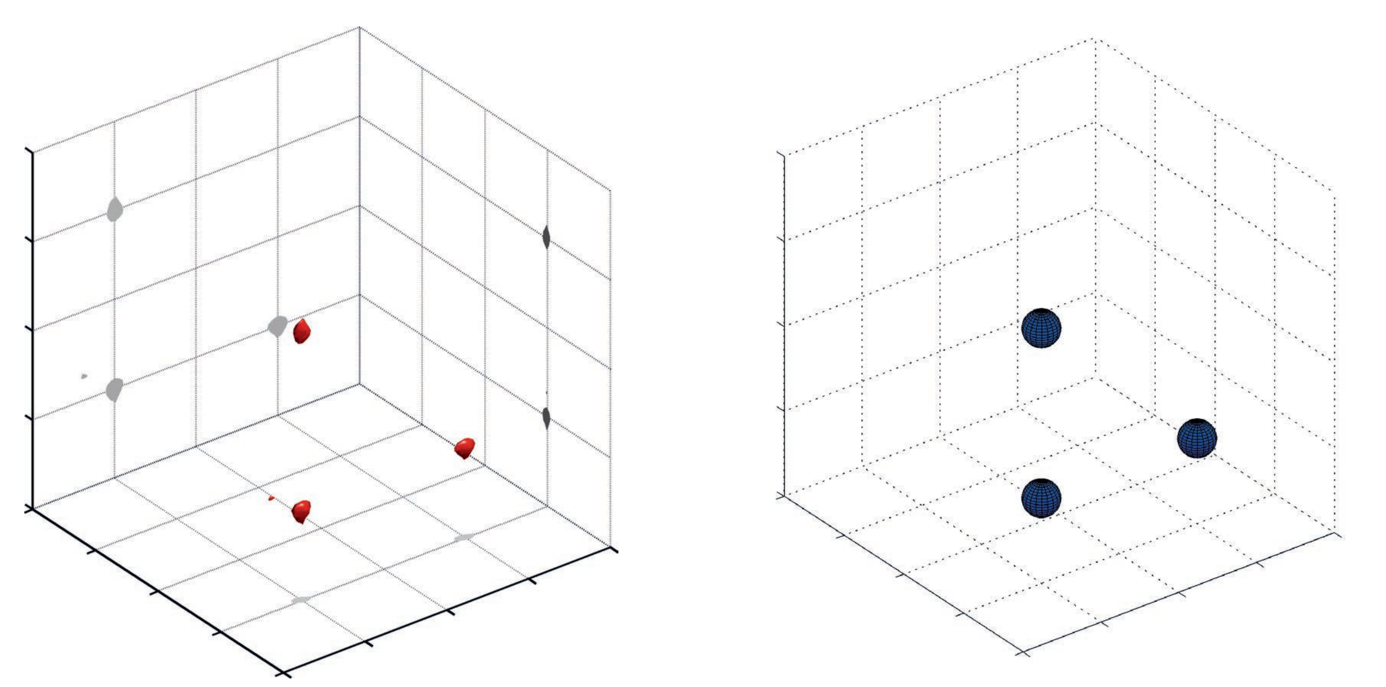
$$Pe^{2ik\mathbf{d}\cdot z} = 0$$

if and only if z is the projection of some z_j onto the plane perpendicular to **e**. If u_k are the left singular functions of F, and there are r non-zero singular values, then

$$\|Pe^{2ik\mathbf{d}\cdot z}\|^2 = \sum_{k=r+1}^{\infty} \left| \int_{\mathbb{S}^1} u_k(\mathbf{d}) e^{-2ik\mathbf{d}\cdot z}\,\mathrm{d}s(\mathbf{d}) \right|^2$$

By plotting the reciprocal of this projection, we are able to find an object's location in the plane orthogonal to e.





By changing the direction from which we image the unknown object, we change the reconstructed image (top row). Combining many directions gives a full three-dimensional reconstruction. (bottom left) True locations are at bottom right.

Quasi-backscattering inversion

The following algorithm describes the quasi-backscattering scheme:

Data Collection

- Choose **e** and place transmitter
- Transmit plane wave toward obstacle
- Collect far field data
- Construct the far field operator

(move transmitter)

Sampling

- Select a grid of sampling points
- Calculate singular value expansion of F
- Pick z in sampling grid
- Calculate the projection operator at z

(move z)

Visualization

- Plot contours of the reciprocal of the projection operator

Three-Dimensional Visualization

- Run experiments with different **e**
- Interpolate results onto 3D sampling grid
- Regularize results[1]
- Average results from different experiments
- Plot 3D contours of averaged results

[1]We regularize using a total variation minimization algorithm which sharpens edges. We also apply a local normalizing effect to the two-dimensional projections.

Conclusions and future work

By using the quasi-backscattering experimental set up, we are able to generate projections of the locations of small obstacles from their far field pattern. In particular, we are able to substantially reduce the amount of data required to do this.

Using similar techniques, we are able to find locations of large spherical obstacles. Furthermore, given two dimensional projections, we are able to approximate the third dimension of an object's location.

In the future, we hope to extend these results to electromagnetic waves. We would also like to use the time-domain aspect of acoustic and electromagnetic waves to improve our ability to locate obstacles.

References

F. Cakoni and D. Colton, A Qualitative Approach to Inverse Scattering Theory. Springer, New York NY (2014).

H. Haddar and J. Rezac, A quasi-backscattering problem for inverse acoustic scattering in the Born regime, Submitted (2015).

Acknowledgements

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