Phytoplankton Aggregations: A Run-and-Tumble Model with Autochemotaxis

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Introduction

Heterosigma akashiwo, a specific species of phytoplankton, is the cause of harmful algal blooms (HABs) in waterways around the world, causing millions of dollars in damage to farmed animals and destroying ecosystems. Developing a fundamental understanding of their movements and interactions through phototaxis and chemotaxis is vital to comprehending why these HABs start to form and how they can be prevented ([1],[2]). We create a complex and biologically authentic mathematical and computational model reflecting the movement of plankton (see Fig. 1), incorporating their run-and-tumble dynamics along with autochemotaxis, in which organisms release the chemical they are attracted to.

Plankton Aggregations

1. Chemical signaling, the depth of the water column, and photosynthesis are key mechanisms for the formation of aggregation structures.
2. We removed the effects of photosynthesis and the depth of the water column by carefully designed experimentation. **Aggregations occur in the absence of both fluid flow and light**.
3. Therefore, we focus on autochemotaxis in the 1D and 2D cases. We explore several choices for this deposition function, $f(c)$ (Fig. 3).

References

[1] Keller, Segel: Model for chemotaxis. *J. Theor. Biol.* 1971;30(2); 225-234.

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[3] Bearon, Pedley: Modelling run-and-tumble chemotaxis in a shear flow. *Bull. Math. Biol.* 2000; 775(62).

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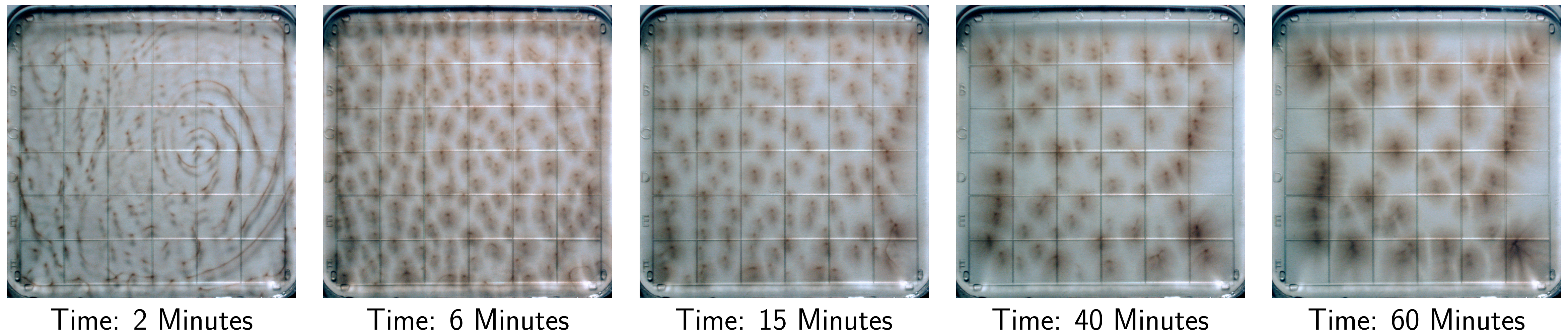

Fig. 1: The evolution of aggregations by *Heterosigma akashiwo* during an hour long experiment with light in shallow water.

The Non-Dimensional Model

1. The chemical, $c(\mathbf{x},t)$, decays and diffuses and the density of plankton, $\rho(\mathbf{x},t)$, deposits the chemical based on an unknown $f(c)$ (where $d_1, d_2 > 0$):
$$c_t = d_1 \Delta c - d_2 c + f(c)\rho.$$
2. Plankton move in a run-and-tumble fashion ([3]) and tumble with the probability,
$$\mathbb{P}\left[\text{Tumb.} \in (t, t+\Delta t)\right] = \frac{1}{2}\left(1 - \frac{\mathbf{e}_\theta \cdot \nabla c}{\sqrt{(\mathbf{e}_\theta \cdot \nabla c)^2 + \delta^2}}\right)\Delta t$$
where $\mathbf{e}_\theta = \langle \cos(\theta), \sin(\theta) \rangle$ and $\delta > 0$.

Numerical Simulations

1. We solved the 1D system numerically:
$$c_t = d_1 c_{xx} - d_2 c + f(c)\rho, \tag{1}$$
$$\rho_{tt} + \rho_t = \rho_{xx} - \frac{\partial}{\partial x}\left(\frac{c_x \rho}{\sqrt{c_x^2 + \delta^2}}\right). \tag{2}$$
Fig. 2 shows the evolution of pattern formation until a non-constant steady state.
2. We simulate a Lagrangian model for the 2D system using a continuous c and utilizing $\approx 10^6$ plankton to approximate the continuous ρ. Fig. 4 shows long-time behavior of ρ for varied $f(c)$.

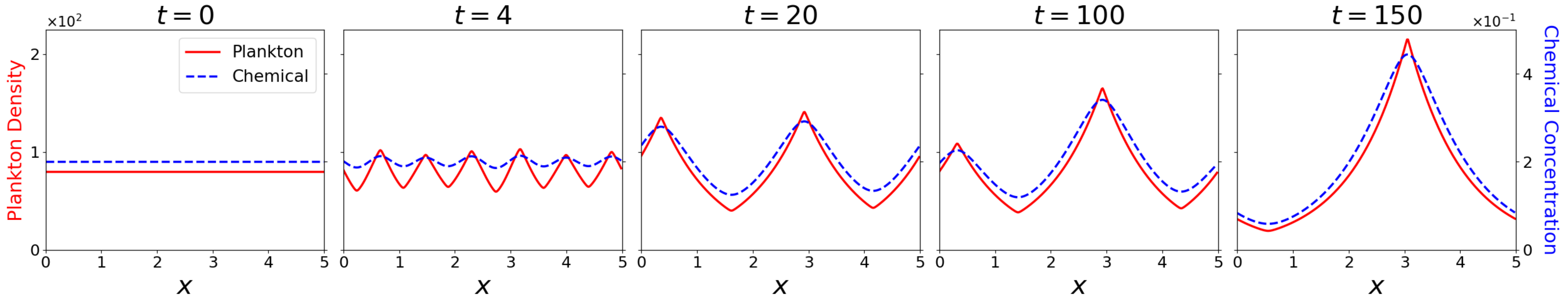

Fig. 2: Numerical solution to (1)-(2) for chemical c and plankton ρ, where $f(c) = f_1(c)$.

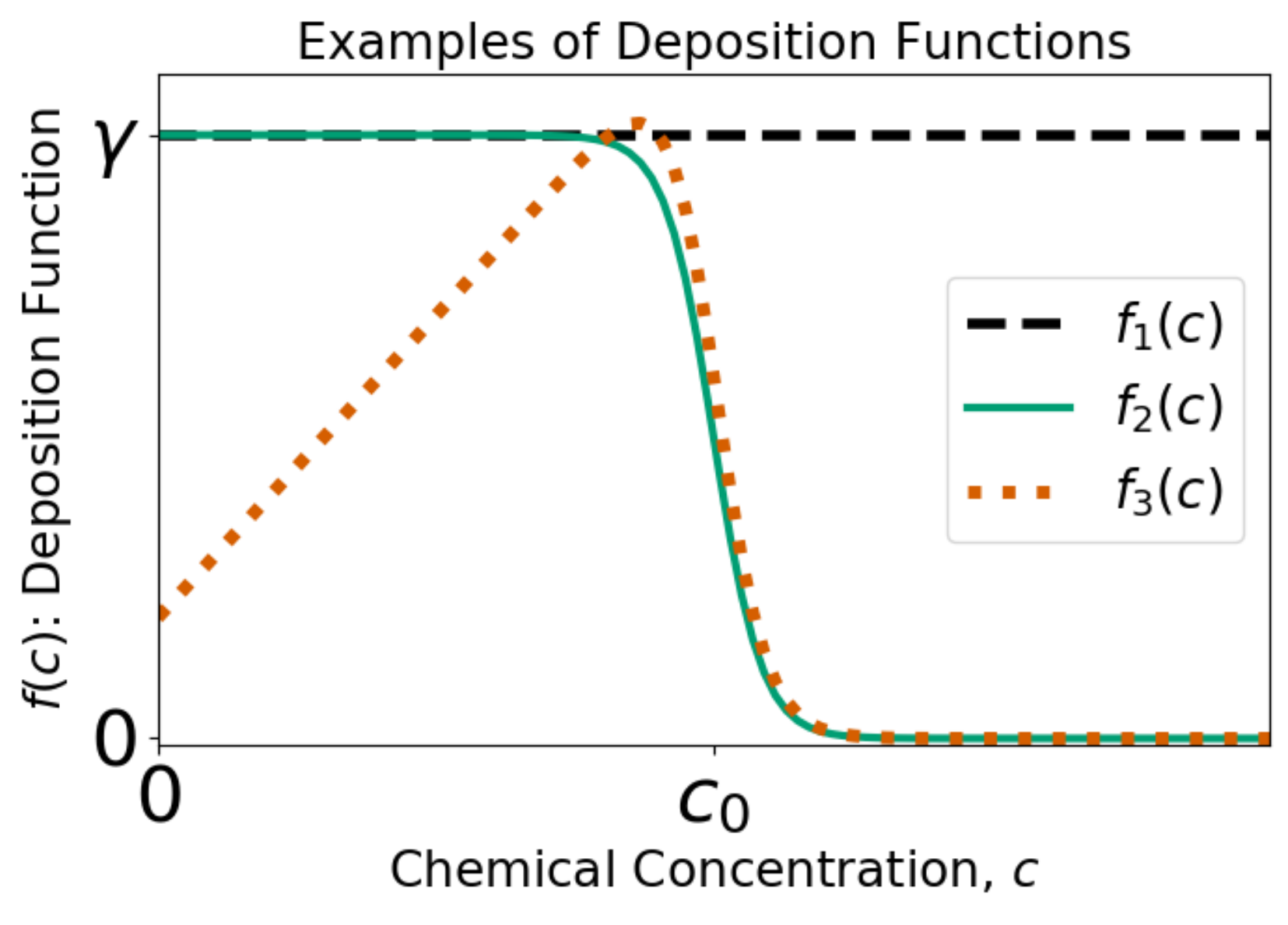

Fig. 3: Deposition functions where $\max f(c) = \gamma$.

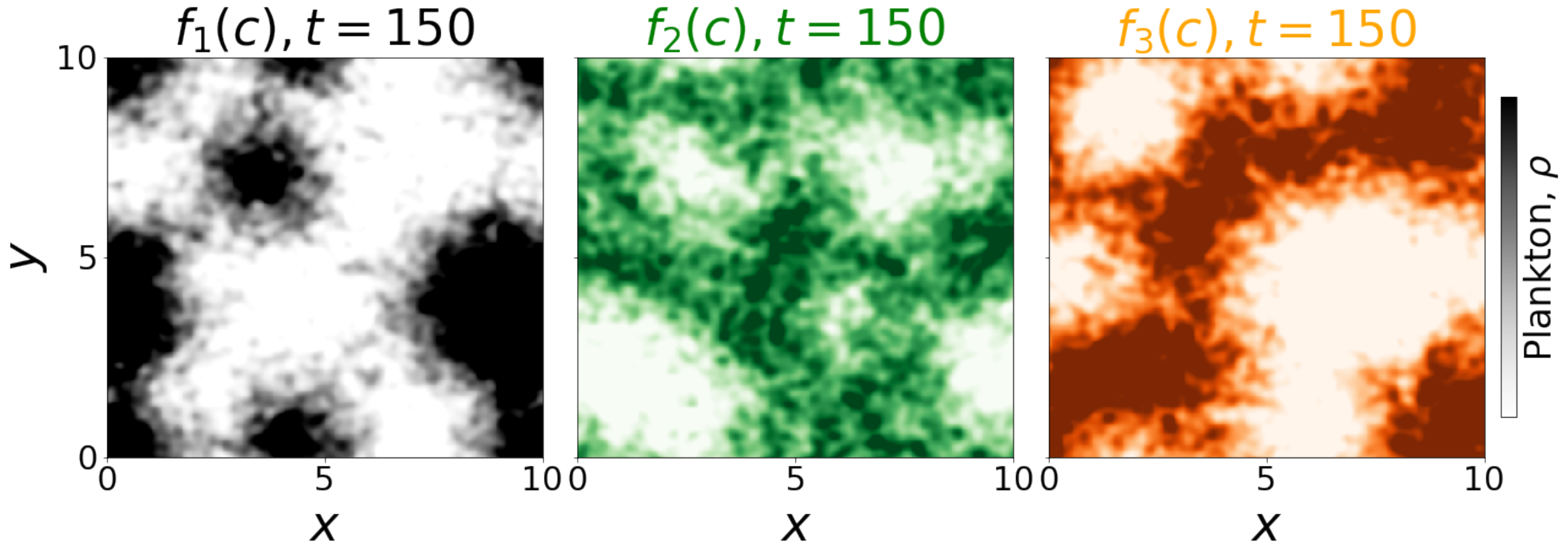

Fig. 4: Long time behavior of ρ in the 2D model for various deposition functions.

Main Achievements

1. Developed a 1D and 2D PDE system that describes the interaction between the chemical field and an ecology of plankton.
2. Conducted Fourier stability analysis on the 1D and 2D systems to establish stability conditions for an aggregation of plankton.
3. Performed 1D and 2D simulations to explore changes in parameters related to run-and-tumble dynamics and autochemotaxis.

Analysis from Simulations

1. Pattern formation is more likely as the chemical diffuses more slowly or decays more rapidly.
2. As $\delta \to 0$, i.e., the plankton's chemotactic sensitivity increases, pattern formation becomes more likely and the aggregations are more dense.
3. Simulations reflect experimentation as we see several aggregations initially form and merging events take longer to complete over time.
4. The deposition functions produce varied results, qualitatively and quantitatively. Both f_1 and f_3 seem to be the closest to mirroring experimental results, but we seek to find the true $f(c)$.

Conclusions and Future Research

- Chemotaxis and deposition play a vital role in aggregations and their stability.
- We will search for a deposition function that accurately describes the process of autochemotaxis.
- We will incorporate density, photosynthesis, vertical migration, optical attenuation, fluid motion, and toxins.

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- Open source code available on GitHub