



Droplet Stability and Coalescence

A problem for the Twenty-Eighth Annual Workshop on
Mathematical Problems in Industry

At the University of Delaware

Mark Hurwitz
June 11, 2012

About Pall Corporation

- Leader in Filtration, Separation and Purification
- Scientific, technology and advanced engineering company
- Founded in 1946
- Global Footprint
- Over 10,000 Employees
- \$2.7B Annual Revenue year ending Aug. 2011



Droplet Stability

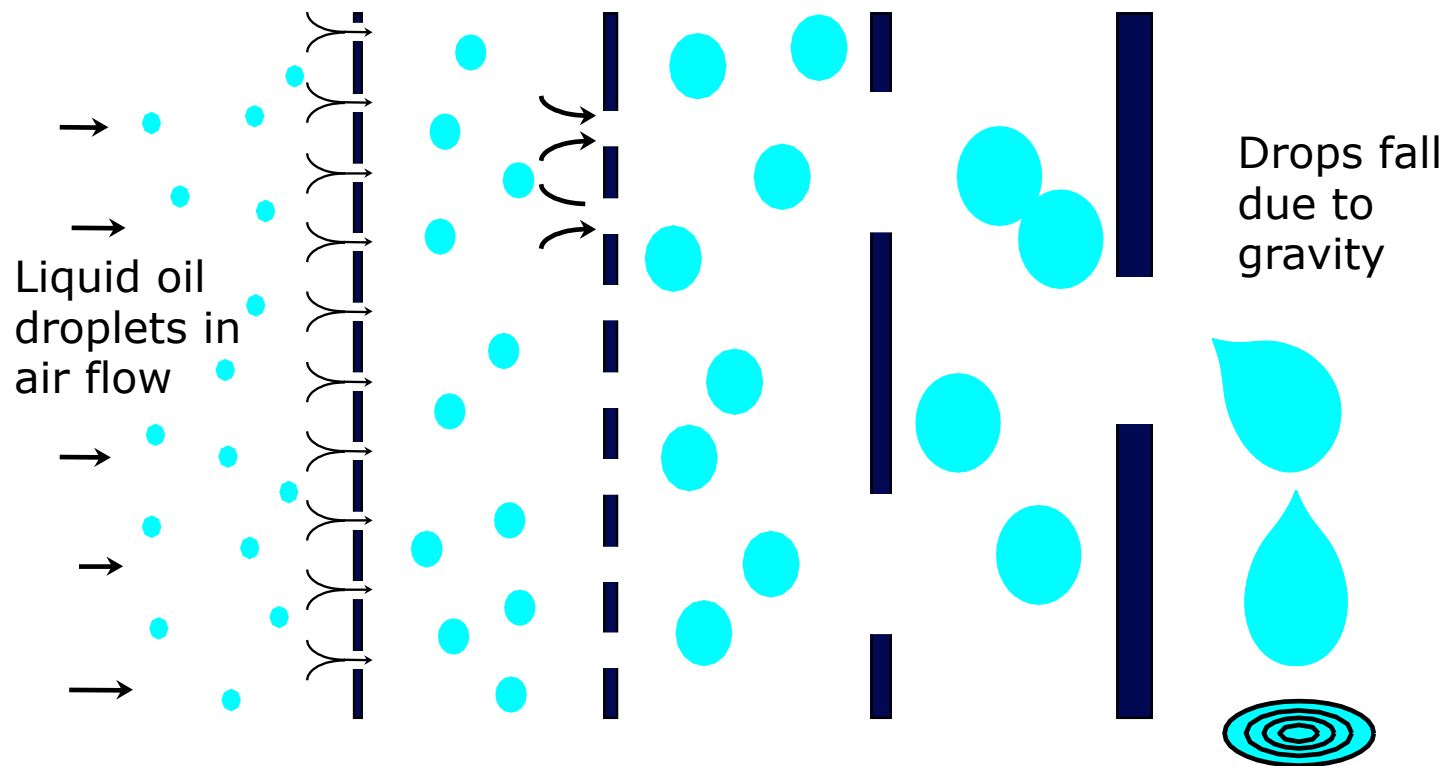
Mark Hurwitz

June 11, 2012

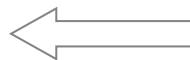


Coalescence: Two or more droplets merge

- Random collisions can cause coalescence
- A coalescer forces droplet interaction in a fibrous structure
- Schematic example:



Big equipment means small improvements are important

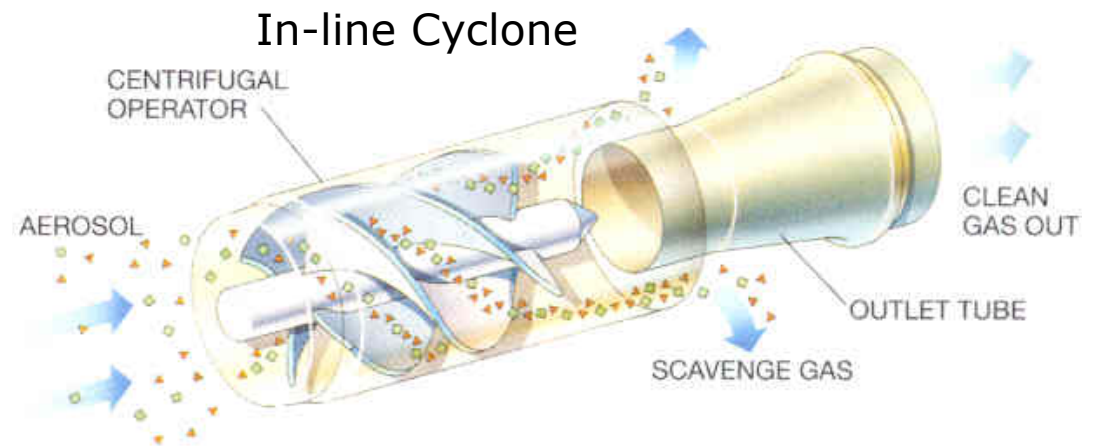
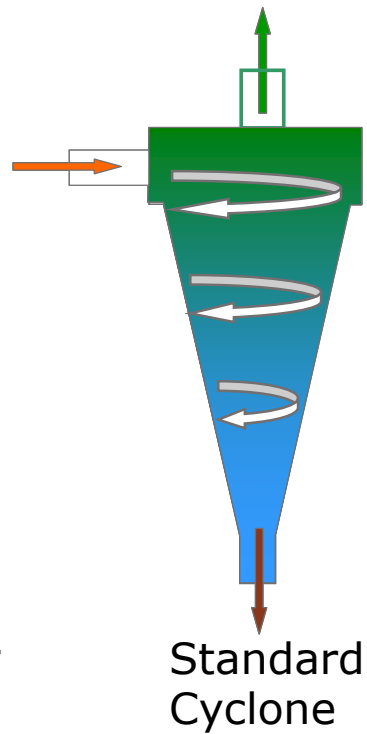
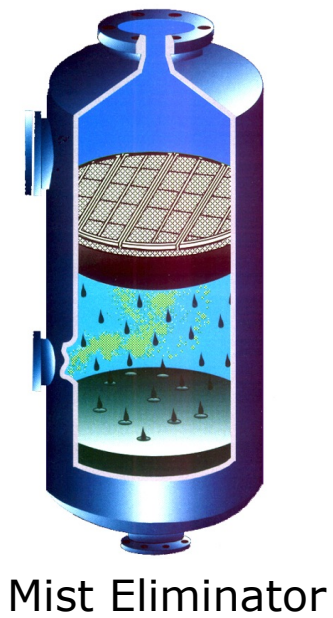
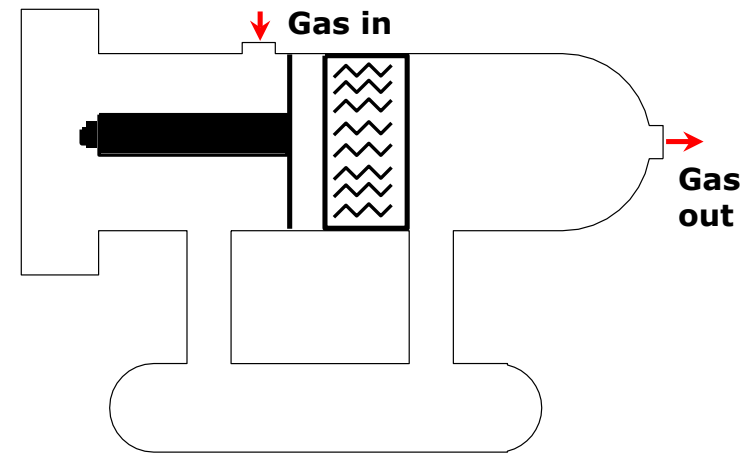
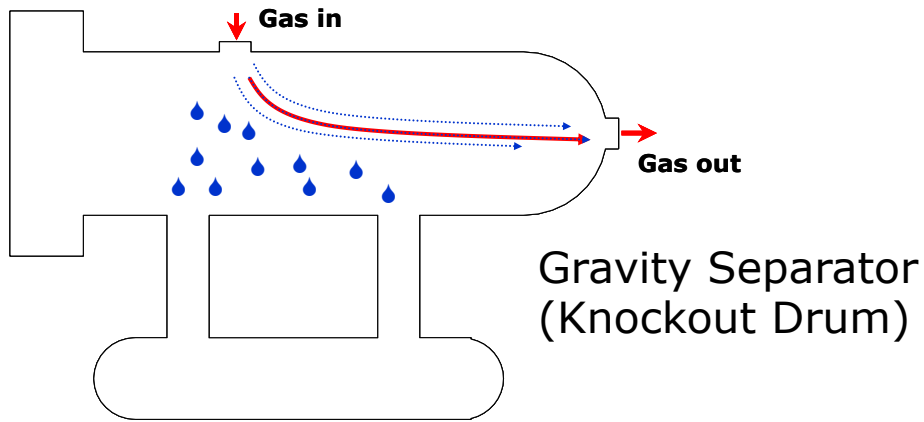


North Sea off-shore gas field.
Removal of salt-containing produced
water before export compressor



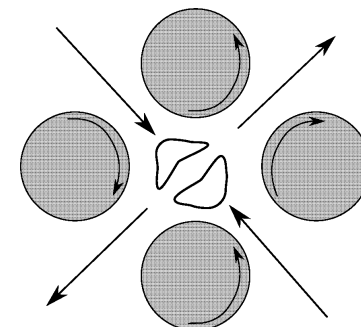
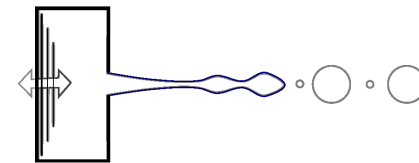
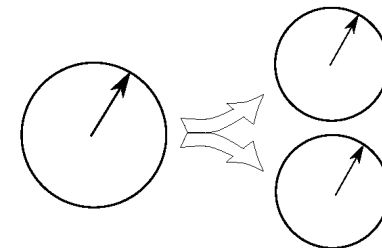
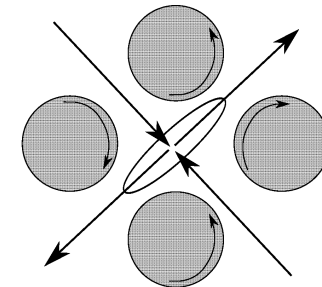
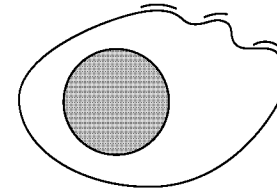
Middle East gas treating plant.
Removal of amine from sweet gas
downstream of contactor, protection
of glycol unit

Other types of liquid / gas separators



Brief (and very select) history of droplet analysis

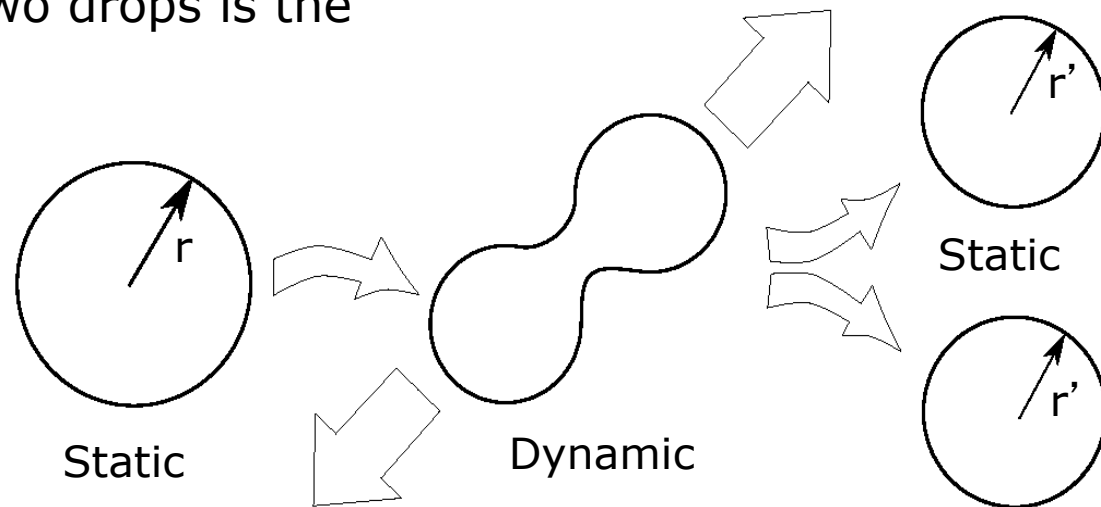
- **1932**, H. Lamb, “Hydrodynamics, 6th edition”
 - Inertia versus gravity for waves and tides
- **1934**, G. I. Taylor, “The Formation of Emulsions in Definable Fields of Flow”
 - Viscosity versus surface energy
 - Emulsions and energy to make small droplets
- **1960**, R. Shinnar & J. M. Church, “Predicting Particle Size in Agitated Dispersions”
 - Surface energy versus turbulence
 - Estimate stable droplet size in turbulent flow
- **1994**, J. Eggers & T. F. Dupont, “Drop formation in a one-dimensional approximation of the Navier-Stokes equation”
 - Surface energy, inertia and viscosity
 - Ink jet printing jet stability and drop size
- **2006**, F. Baldessari & L. G. Leal, “Effect of overall drop deformation on flow-induced coalescence at low capillary numbers”
 - Coalescence of viscous droplets in shear flow



Problem 1: What makes an inviscid drop be stable?

- Reasonable starting point:
 - Radius: $r = 0.1$ to 10 cm
 - Constant density: $\rho = 1.0$ gm/c.c.
 - Surface tension: $\sigma = 70$ dyne/cm
 - Neglect viscosity and evaporation

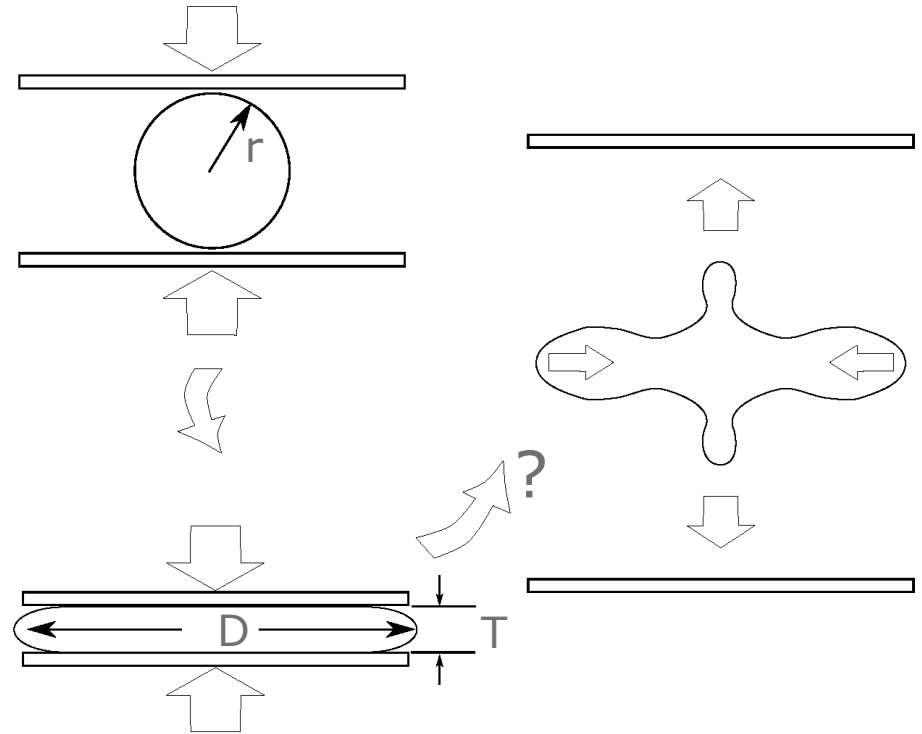
- Minimum energy to make two drops is the change in surface energy
- Is this a useful criterion?



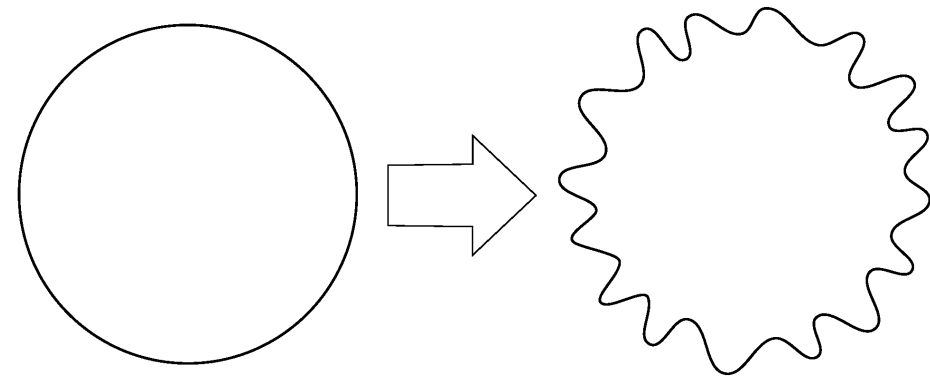
- Volume conservation:
 - $r' = r / 2^{1/3}$
- Change in surface energy:
 - $\Delta E = 4\pi\sigma (2r'^2 - r^2) = 1.04\pi\sigma r^2$
 - Area increase = 25%

Surface area extremes

- Flatten droplet with parallel, non wetting plates
 - 25% increased area: $T/2r \sim 0.49$
 - Is this enough to expect instability?

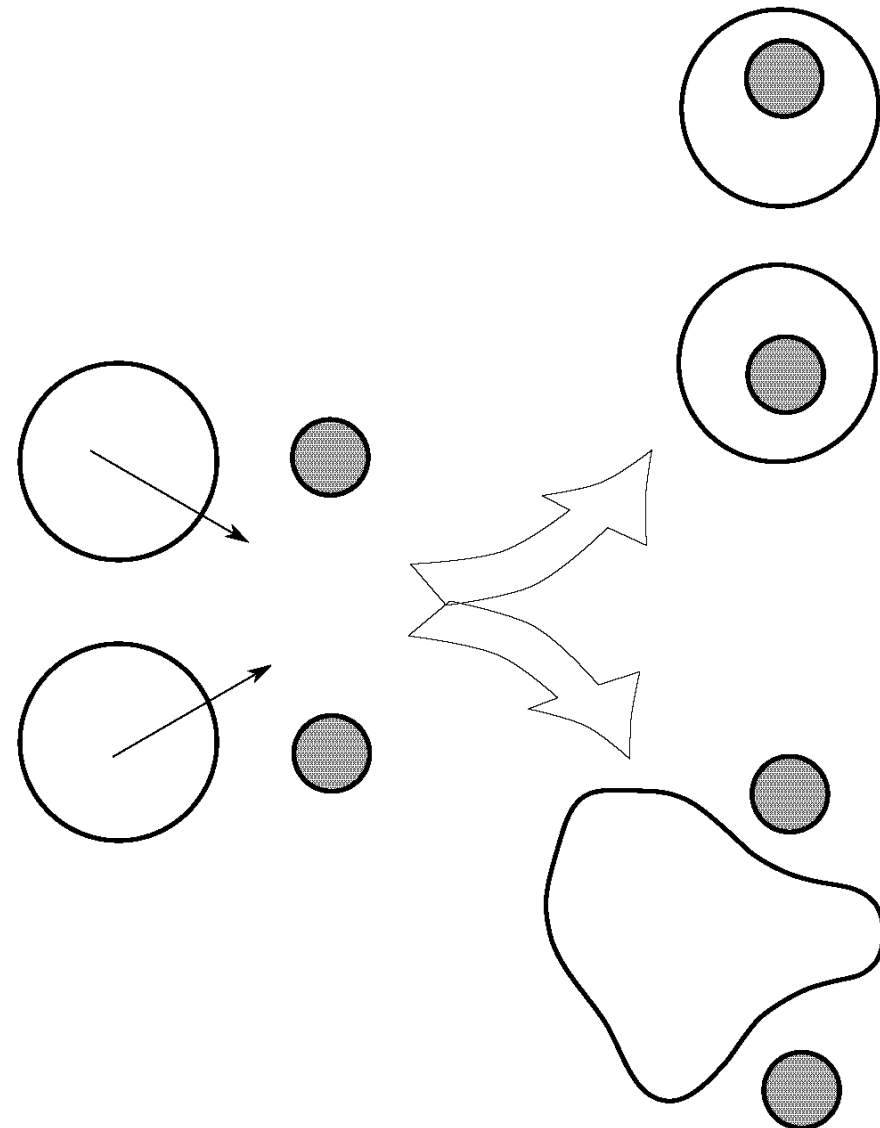


- Short wavelength pressure fluctuations
 - Large area increase with small change in aspect ratio
 - What are the resonances?



Problem 2: Lubrication layer, phobic and phylic fibers

- (In case problem 1 is too easy!)
- Lubrication layer tends to keep droplets separate and separate from fibers
- Droplets wet and stick to phylic fibers
- Phobic fibers push droplets together
- What are the criteria that predict coalescence?



Thank you.
And now the fun begins!