Constructions of Normal and Non-Normal Cayley Graphs for Isomorphic Regular Groups

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♢ CONTENTS

- Introduce normal and nonnormal Cayley Graphs for isomorphic regular groups.
- Look at Cartesian, Direct, Strong products of the graphs.
- Construct new NNN-graphs of non-prime power order.

♢ Cayley Graphs

Let G be a group, and $\emptyset \neq S \subset G$: $S^{-1} = S$ and $\mathbf{1} \notin S$.
$\Gamma = Cay(G, S)$ is a Cayley graph: $V(\Gamma) = G$; $E(\Gamma) = \{(x, y) | xy^{-1} \in S\}$.
We say G: *defining group*, S: *connection set/generating set*

1. Γ is connected if and only if S generates G.
2. $\hat{G} \trianglelefteq Aut(\Gamma)$, $\hat{G} \cong G$
   $\hat{G} = \{\hat{g} | g \in G\}$: for $g \in G$

$$\hat{g} : x \to xg, \text{ for all } x \in G.$$

Cayley graphs characterization

A graph is a Cayley graph if and only if its automorphism group contains a subgroup acting regularly on the vertex set of the Cayley graph.

Spiga: $\Gamma = \Gamma_d$, $2^{d^2/64-(d/2)\log_2(d/2)}$ regular subgroups.

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◇ AUTOMORPHISMS

Let $A = Aut(\Gamma)$.

Definition (M. Y. Xu., 1998)

Γ is a normal Cayley graph for G if $A = N_A(\hat{G})$; otherwise, Γ is nonnormal for G.

$N_A(\hat{G}) = \hat{G} \rtimes Aut(G, S)$,
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Theorem 1 (Wang., Wang., & M. Y. Xu., 1998)

Every finite group other than $Z_4 \times Z_2$ and $Q_8 \times Z_2^m$ with $m \geq 0$, has at least one normal Cayley graph.

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◇ Normal Cayley Graphs

Question (Feng. & Dobson.): Is it possible for a graph to be both a normal and a nonnormal Cayley graph for two isomorphic regular groups?

Some Cayley graphs with exactly one (conjugacy class of) regular subgroup.

These graphs are not Cayley graphs that are both normal and non-normal for two isomorphic regular groups.

- G: CI-Groups
  - $Cay(G,S)$ is a *CI-graph* if all regular subgroups isomorphic to G are conjugate.
  - G is a *CI-group* if all Cayley graphs for G are CI-graphs.
    - $Z_8, Z_9, Z_n, Z_{2n}, Z_{4n}$, n is odd and square-free (Muzychuk.);
    - Q_8, Z_p^2, Z_p^3, p is a prime (Dobson., Godsil., Xu.);
    - D_{2p}, F_{3p}, the Frobenius group, p is a prime (Babai., Li.).
- Any Cayley graph of Z_p except K_p, p is a prime.
- GRRs: $Aut(\Gamma) = G$.

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Definition

An NNN-graph is a Cayley graph that is normal and nonnormal for two isomorphic regular groups.

- BG-graphs: Point graphs of the generalised quadrangles $Q(q-1, q+1)$ with $q = p^k$ and $p \geq 5$. (Bamberg. & Giudici., 2011)
- A strongly regular Cayley graph of valency 35 for the group Z_2^6. $A = Z_2^6 \rtimes S_8$. (Royle., 2008)
- A strongly regular Cayley graph of Z_6^2 with some particular adjacency matrix. $A = Z_6^2 \rtimes Z_2^2$, normal for $Z_3^2 \rtimes Z_2^2$. (Giudici. & Smith., 2010).

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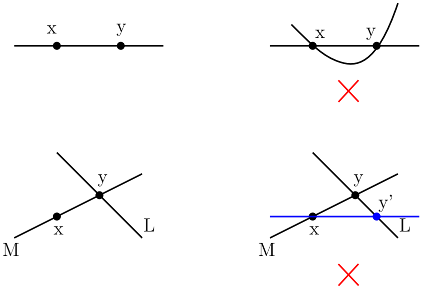
♢ BG-GRAPHS

Let Q be a *generalised quadrangle* of order (s,t) with a point set $\mathcal{P}$ and a line set $\mathcal{L}$.

Q satisfies the following GQ-axioms:

- Each line has $s+1$ points; each point is on $t+1$ lines, and any two points lie on at most one line;
- For each point x not on a line L, there is a unique line M and a unique point y such that x is on M, and y on M and L.



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The *point graph* of Q is the graph having $\mathcal{P}$ as its vertex set, and two vertices x, y are adjacent if and only if they lie on the same line.

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Γ and Q have the same automorphism group.

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A Classical GQ $Q = W(3, q)$:

V: 4-dimensional v.s., with an alternating form f

$$f(u, v) = u_1 v_4 - v_1 u_4 + u_2 v_3 - v_2 u_3.$$

- points: 1-dimensional totally isotropic subspaces,
- lines: 2-dimensional totally isotropic subspaces,
- order is (q, q), $Aut(Q)$ =$\mathrm{P\Gamma Sp}(4, q)$, and $\mathrm{Sp}(4, q) \leq Aut(Q)$.

Payne derived Q^x from $Q = W(3, q)$: Let $x = (1, 0, 0, 0)$.

- $\mathcal{P}_{Q^x}$: points in Q not collinear with x,
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Two Isomorphic Subgroups in $Aut(Q)$:

- elation subgroup $E = \{M_{a,b,c} | a, b, c \in GF(q)\}$ where

$$M_{a,b,c} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -c & 1 & 0 & 0 \\ b & 0 & 1 & 0 \\ a & b & c & 1 \end{pmatrix}$$

- $P = \langle R, \theta_{\alpha_1}, \ldots, \theta_{\alpha_t} \rangle$, where $R = \{M_{a,b,0} | a, b \in GF(q)\}$, and

$$\theta_{\alpha_i} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\alpha_i & 1 & 0 & 0 \\ -\alpha_i^2 & \alpha_i & 1 & 0 \\ 0 & 0 & \alpha_i & 1 \end{pmatrix}$$

♢ BG-GRAPHS

Let $q = p^k$ and $p \geq 5$.

E, P act regularly on the points in Q^x, and $E \trianglelefteq Aut(Q^x)$ while P is not.

Let Γ be the point graph of Q^x (*BG-graph*).

Theorem 1 (Bamberg. & Giudici., 2011)

Γ is a Cayley graph of order q^3 with $q \geq 5^k$, and $Aut(\Gamma)$ contains two regular subgroups E, P, where $E \lhd Aut(\Gamma)$, $P \ntriangleleft Aut(\Gamma)$ and $E \cong P$.

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♢ NNN-GRAPHS

A Strongly Regular Cayley Graph Γ for Z_2^6:

- $v \in V(\Gamma)$:
  - $v \in \{-1, 1\}^8$ with $0, 2, 4$ '-1's;
  - $v_1 = 1$ if it has four '-1's.
- $\{u, v\} \in E(\Gamma) : u \cdot v = 0$.

♢ NNN-GRAPHS

A Strongly Regular Cayley Graph Γ for Z_2^6:

- $|V(\Gamma)| = 64$;
- Γ is $(64, 35, 18, 20)$ strongly regular graph;
- $Aut(\Gamma) = Z_2^6 \rtimes S_8$.

Theorem (Royle., 2008)

Γ is an NNN-graph for Z_2^6.

Γ is non-normal for $Z_2^3 \times Z_2^3$.

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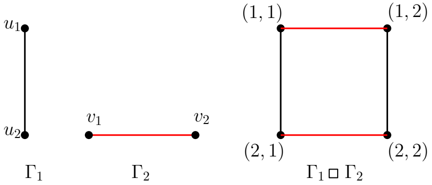
◇ Three Fundamental Graph Products

For $i = 1, 2$, let $\Gamma_i = (V_i, E_i)$ be two finite simple graphs.

Cartesian Product

The Cartesian Product $\Sigma = \Gamma_1 \square \Gamma_2$ is the graph with vertex set $V_1 \times V_2$ such that $\{(a_1, a_2), (b_1, b_2)\}$ is an edge if and only if either $\{a_1, b_1\} \in E_1$ and $a_2 = b_2$, or $\{a_2, b_2\} \in E_2$ and $a_1 = b_1$.

e.g.



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Direct Product

The Direct product $\Sigma = \Gamma_1 \times \Gamma_2$ is the digraph with vertex set $V_1 \times V_2$ such that $\{(a_1, a_2), (b_1, b_2)\}$ is an edge if and only if $\{a_1, b_1\} \in E_1$ and $\{a_2, b_2\} \in E_2$.

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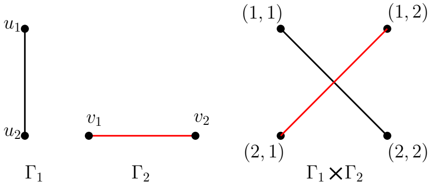

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Strong Product

The Strong product $\Sigma = \Gamma_1 \boxtimes \Gamma_2$ is the digraph with vertex set $V_1 \times V_2$ such that $\{(a_1, a_2), (b_1, b_2)\}$ is an edge if and only if $\{a_i, b_i\} \in E_i$ or $a_i = b_i$ for $1 \leq i \leq 2$.

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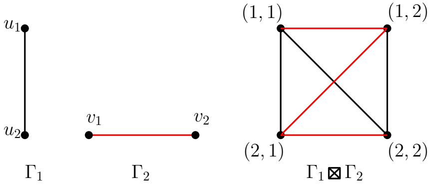

Prime graph: not representable as any of these three standard graph products of nontrivial graphs.

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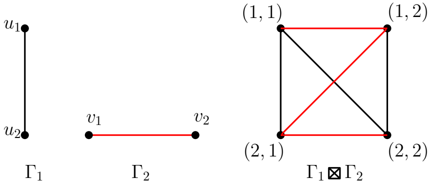

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♢ Automorphisms of Graph Products

Theorem 2 (Imrich. etc.)

Let $\Gamma_1, \ldots, \Gamma_t$ be prime graphs. Let

$$H = (Aut(\Gamma_{k_1}) \wr S_{n_1}) \times \cdots \times (Aut(\Gamma_{k_r}) \wr S_{n_r}), \tag{1.1}$$

where $\sum_{i=1}^{r} n_i = t$ and n_i is the number of factors isomorphic to Γ_{k_i}. Then

1. $Aut(\Gamma_1 \Box \cdots \Box \Gamma_t) = H$;
2. if $\Gamma_1 \times \cdots \times \Gamma_t$ is R-thin and non-bipartite, then $Aut(\Gamma_1 \times \cdots \times \Gamma_t) = H$;
3. if $\Gamma_1 \boxtimes \cdots \boxtimes \Gamma_t$ is S-thin, then $Aut(\Gamma_1 \boxtimes \cdots \boxtimes \Gamma_t) = H$.

R-thin: no two vertices $x, y \in V(\Sigma)$ such that $N(x) = N(y)$.
S-thin: no two vertices $x, y \in V(\Sigma)$ such that $N[x] = N[y]$.

♢ Construct New NNN-graphs

Theorem 3 (Y. Xu., 2017)

Let $\Gamma_1, \ldots, \Gamma_t$ be prime Cayley graphs where Γ_1 is NNN and Γ_i is normal with $i \geq 2$. Suppose Σ is one of the following three types:

(i) $\Sigma = \Gamma_1 \square \cdots \square \Gamma_t$;

(ii) $\Sigma = \Gamma_1 \times \cdots \times \Gamma_t$, and Σ is non-bipartite and R-thin;

(iii) $\Sigma = \Gamma_1 \boxtimes \cdots \boxtimes \Gamma_t$, and Σ is S-thin.

Then Σ is an NNN-graph.

♢ Construct New NNN-graphs

Γ: point graph of Q^x, $x \in W(3,q)$, $Aut(Q^x)$ $=$P$\Gamma S_p(4,q)_x$ when $p \geq 5$; $W(3,q)$ is *elation GQ*.

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Let $\Gamma_1, \ldots, \Gamma_t$ be BG-graphs. Then the graph obtained by taking one of the three standard products of $\Gamma_1, \ldots, \Gamma_t$ is an NNN-graph.

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Lemma 4

Γ is a prime graph.

Let $\Sigma = K_2$, Σ is a normal circulant for Z_2.

Main Theorem 2

There is an NNN-graph for Z_2^m if and only if $m \geq 6$.

Remark: Z_2^m is a CI-group when $m \leq 5$.

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