Newsletter of the **Department of Mathematical Sciences** @ University of Delaware

RECKONINGS

SUMMER **2020**

Inside

Message from the Chair

LOUIS F. ROSSI

Dear Alumni and Friends,

You might be surprised to receive a message of pride, hope and gratitude from me in the middle of a pandemic and at a time when some of the systemic inequities in our society, especially those affecting Black students, are being laid bare. These crises are having a profound impact on our students and our university, and we are facing these challenges today, tomorrow and in the days to come. There is no better indicator of the quality and strength of our students, faculty and our department than our response to these crises, and I can say with pride that the Mathematical Sciences faculty spared no expense and no effort to meet the needs of our students. As I entered the last year of my term, I expected this final column would provide a broad view of the last five years. However, one of the first things I learned as a new Chair was that there would be times when I would not get to choose my priorities. Significant problems and challenges can arrive without warning and upend everything else. The abrupt arrival of COVID-19 to our campus was one of those times.

It's an exciting story to tell. There is not enough space to tell all of it here, but I will share some of the highlights. While some of our faculty have experience teaching select courses online, all faculty dropped everything they were doing, rose to the challenge and transformed every single course in our catalog into an online course, and we did it in less than two weeks. The faculty did not do it alone. Our grad students, who run our recitations and lab sections, dove in as well. The challenges are more profound than most fully appreciate. Engagement is the cornerstone of the best education, so distancing instructors from their students, no matter how good the technology is, is like asking surgeons to operate without using their hands. However difficult our circumstances were, we never forgot that things were harder for our students.

Logistically, iPads, stylus' and webcams were in high demand. Supply chains for online teaching equipment were under a lot of pressure, so our faculty and grad students scoured local retailers and bought what they could and made the best of what they could get. On the software side, faculty mastered new ways to deliver lessons, engage their students and measure what they were learning. And, of course, many of our faculty are parents too, so faculty households were sending and receiving online courses all day and night. At the same time, we knew the pandemic would impact some of our students disproportionately. Some lacked equipment at home. Some lacked broadband at home. Some did not have spaces to work at home or had other very challenging situations. These are not easy things for students to talk about with their professors, so our faculty had to double as trusted counselors and advisors. We couldn't solve every problem, but we kept trying. We took every laptop out of our teaching labs, re-imaged them, and distributed them as loaners to students who didn't have adequate equipment at home.

While we are not out of the woods yet, I am hopeful about the future for two important reasons. First, as a unit, we have gained considerable expertise from the exercise of going online, and so, while we look forward to going back to the classroom when the pandemic is over, I expect most faculty will be using more technology, even if it is something as simple as recorded mini-lectures or Zoom office hours, to supplement classroom experiences. Second, the pandemic has tested the strength of our organization and has provided us with a unique opportunity for renewal when we return to our beautiful campus with all its learning spaces. We know

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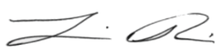
what works well. We also know where we can do better, and we will!

Now, for a few words of gratitude for all the ground we have covered in the last five years. Working together as a department, we have made Ewing a much better place to spend time than it used to be. We have good wifi coverage throughout the building. Mathematics is collaborative and collaborative spaces are crucial. With the help of alumni and friends, we updated our undergraduate lounge and the faculty/graduate student lounge. The coffee is better. We also have a great seminar room and a real collaborative working space in the building now. We are able to support more student research experiences over the summer. When undergraduate or graduate students have papers accepted at professional conferences, generous donations from our alumni and friends support their travel so that they can present their work.

Over the last five years, some of the most important investments we have made are in our faculty. We've hired 12 new permanent faculty, and it's a more diverse cohort than we have ever hired before. The new faculty have brought new research and teaching expertise that have had a profound impact on campus and in the mathematical sciences community. We're bringing mathematics to our students in new and exciting ways, and it's thrilling to hear students casually say on campus that their required math class is one of their favorites. We're also paying it forward by training our young math educators in active learning methodologies. The number of students on campus who choose to major in mathematics is at an all time high. We launched a new learning center and a research center. We've seen many of our faculty recognized with teaching, research and service awards. We saw the birth of a new undergraduate degree program in Applied Mathematics, and the explosive growth of an existing one, Actuarial Sciences. We were instrumental in launching a new cross-cutting masters program in data science. Outside the department, some of our faculty are excited to be playing roles in new campus-wide initiatives in data science, neuroscience and quantum science. We remain a regional powerhouse with a multitude of active seminar series. Many of our faculty are asked to serve on the editorial boards of top journals in their fields. We routinely host important mathematical conferences. I'm happy that one of my colleagues is returning from the National Science Foundation after serving a term as a program officer while another colleague will be starting a term as one, and many of our faculty serve on national funding review panels. While these efforts involve everyone in the department, few of these achievements would have been possible without the vitality that our new faculty have brought to the department.

I will end my message of pride, hope and gratitude here. Serving as Chair has been a job like no other, and it's been a pleasure and privilege to serve this department in this capacity for the last five years. There are many challenges ahead for the faculty and students of Mathematical Sciences, as we work through this pandemic and take positive social steps to build a more diverse and welcoming community. I look forward to helping the department face these challenges as a member of the faculty under the leadership of our incoming Chair, Mark Gockenbach, who we will welcome to UD this coming Fall.

Stay smart. Stay safe. Stay healthy.



Louis Rossi
Professor and Chair

Non-Local Games – A Glance

IVAN TODOROV

1. NO-SIGNALLING CORRELATIONS AND ENTANGLEMENT

Quantum entanglement, allowed by the postulates of Quantum Mechanics, is probably one of the most counter-intuitive quantum-physical phenomena. It is well-known that leading physicists of the time were dissatisfied with the – new, back then – theory because of entanglement's empirical consequences, and proposed alternative ways to explain the "spooky action at a distance" triggered by it. At the same time, from a mathematical perspective, entanglement has a neat and simple description, which at its core translates into the fact that not every positive semi-definite complex matrix of size $n^2 \times n^2$ is a linear combination, with positive coefficients, of tensor (Kronecker) products of positive semi-definite matrices, each of size $n \times n$. The debate was resolved in 1964 by Bell who demonstrated that no "classical" physical theory exists that captures the consequences of entanglement observed in experiments.

From a mathematical viewpoint, Bell's Theorem states that two convex subsets, C_{loc} (the *local class*) and C_q (the *quantum class*), of the set of all conditional probability distributions $p = \{(p(a, b|x, y))_{a,b\in A} : x, y \in X\}$ are distincts. Here, X and A are finite sets, and the conditional probability distributions are *no-signalling*, that is, the marginal distributions $(p(a|x))_{a\in A}$, $x \in X$, and $(p(b|y))_{b\in A}$, $y \in X$, are well-defined. The interpretation of such p is the following: a *referee* conducts an experiment, in which she sends input x to Alice and input y to Bob. Alice and Bob respond with outputs a and b, respectively. The value $p(a, b|x, y)$ is the probability that Alice and Bob spit out the output pair (a, b), given the input pair (x, y). The no-signalling requirement is a mathematical formulation of the fact that Alice and Bob do not communicate during the experiment. The statistics of Alice and Bob outputs, which follows the distributions p, describe their mutual correlation. The term *no-signalling correlation*, adopted for naming p, thus becomes natural.

2. NON-LOCAL GAMES

In a *non-local game*, the referee sends inputs (x, y) to the two players Alice and Bob and expects their output pair (a, b) to satisfy certain *rules*; the rules are given by a function $\lambda : X \times X \times A \times A \to \{0, 1\}$, and the players *win against the referee* if $\lambda(x, y, a, b) = 1$ (and lose otherwise). Alice and Bob may decide to employ a *deterministic strategy* – a pair of functions $f : X \to A$ and $g : Y \to B$, leading to the $(f(x), g(y))$-output given the (x, y)-input, or a *probabilistic* one – a no-signalling correlation p. (Of course, a deterministic strategy can be viewed as a probabilistic one by letting $p(a, b|x, y) = 1$ if and only if $a = f(x)$ and $b = g(y)$.) A probabilistic strategy is *perfect* if it disallows "wrong" responses:

$$\lambda(x, y, a, b) = 0 \Longrightarrow p(a, b|x, y) = 0.$$

For example, in the *graph colouring* game, the inputs are pairs (x, y) of vertices of a fixed graph G, the outputs – pairs (a, b) of "colours", and the rules capture the natural conditions that the players are in the possession of a proper colouring of G:

- if $x = y$ then the players should respond with $a = b$;
- if $x \sim y$ then they should respond with $a \neq b$.

We are assuming that the referee is memoryless: the players may vary their responses to the same input pair over the course of the game. It becomes intuitively plausible that, in such a scenario, they may be able to colour the graph G probabilistically with *strictly smaller* number of colours than the classical chromatic number $\chi(G)$ of G. This is indeed the case: the Hadamard graph (vertex set $\{-1, 1\}^n$, adjacency $x \sim y \Leftrightarrow x \cdot y = 0$) has a *quantum chromatic number* (the smallest number of colours for which a perfect strategy from the class C_q exists), strictly smaller than $\chi(G)$.

3. THE CORRELATION TOWER

The correlations from the class C_q have a simple mathematical definition:

$$p(a, b|x, y) = \langle (E_{x,a} \otimes F_{y,b})\xi, \xi \rangle,$$

where ξ is a vector of norm one in some finite dimensional Hilbert space H, and $(E_{x,a})_{a\in A}$ and $(F_{y,b})_{b\in A}$ are positive operator valued measures (POVM's) on H for all $x, y \in X$ (that is, $\sum_{a\in A} E_{x,a} = I_H$ for each x, and similarly for the F's). The set C_{loc} is contained in C_q: its elements are the convex combinations of independent probability distributions $p_A(a|x)p_B(b|y)$. And the postulates of the relativistic model of Quantum Mechanics allow for a formally larger set C_{qc}: $p(a, b|x, y) = \langle E_{x,a}F_{y,b}\xi, \xi\rangle$, where $E_{x,a}$ and $F_{y,b}$ act on the same Hilbert space, which is now allowed to be infinite dimensional, and the commutations $E_{x,a}F_{y,b} = F_{y,b}E_{x,a}$ hold. Letting C_{ns} denote the set of all no-signalling correlations, we have the inclusion chain

$$C_{loc} \subseteq C_q \subseteq C_{qc} \subseteq C_{ns}$$

Until recently, it was an open problem if the set C_q is closed – now resolved in the negative by Slofstra, and whether the closure C_{qa} of C_q coincides with C_{qc}. The latter equality became known as the *Tsirelson problem* – an important question when viewed from a foundational perspective, as it asks if the two models of Quantum Mechanics – relativistic and non-relativistic – are equivalent. As recently as 2020, a negative solution by Ji-Natarajan-Vidick-Wright-Yuen came to light. This solution simultaneously settled an old and difficult problem in operator algebras, the *Connes Embedding Problem*, known to be equivalent to Tsirelson's problem, asking if the approximately finite dimensional and finite von Neumann factor is large enough for its ultrapower to "contain" any other (separably acting) finite type II factor.

4. CLOSING DOORS OR OPENING WINDOWS?

Given the described progress, are there any – still – outstanding meaningful and interesting questions in this mathematics quarter? Not surprisingly, the answer is affirmative. The distinction between the relativistic and the non-relativistic models gives a new impetus to study non-local games employing advanced operator algebraic methods. In the course of the past decade, a plethora of new operator algebras, designed from games, was introduced; their exploration is only at the very start. Graph colourings discussed in Section 2 is just an example of a more general class of *graph homomorphism games*, where two graphs G and H are given, and the players aim at convincing the referee that they possess a graph homomorphism from G to H. The latter game, on the other hand, is just an example of a vast family of games, including *synchronous games, variable assignment games, unique games,* and *imitation games*. The last two are important in theoretical computer science.

Another exciting direction is pointed to us (again) by non-commutativity. A quantum version of graphs was introduced in 2013 by Duan-Severini-Winter. Much in the vein of non-commutative geometry, given a graph G on d vertices, they considered the space $\mathcal{S}_G$ of matrices in M_d, supported on the edges of G *(see Figures 1 and 2)*. The space $\mathcal{S}_G$ is an *operator system* – it is closed under taking conjugate transposes and contains the identity matrix – and "remembers" the isomorphism class of G. Graph theoretical questions about G may thus be reduced to linear algebraic questions about $\mathcal{S}_G$. What is even more important from the point of view of quantum information theory, general operator systems in M_d – *non-commutative graphs* – cary important information about quantum channels, including their zero-error codes, codes with side information and private subsystems. *Non-commutative combinatorics,* thus arising, unveils routes for quantization of classical graph parameters and concepts with immediate relevance in quantum information theory. There are, for example, a non-commutative Ramsey Theorem, non-commutative chromatic and fractional chromatic numbers, Lovász theta numbers, Shannon capacity, and many more. And although one cannot see the vertices or the edges of a non-commutative graph, one can still define non-commutative graph homomorphisms. This points to the possibility for extending the realm of non-local games into the purely quantum world, where inputs and outputs are no longer classical – no longer symbols taken from finite lists X and A – but quantum states of finite – or even infinite! – quantum systems.

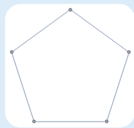


Figure 1. The cycle C_5.

$$\mathcal{S}_{C_5} = \begin{pmatrix} * & * & 0 & 0 & * \\ * & * & * & 0 & 0 \\ 0 & * & * & * & 0 \\ 0 & 0 & * & * & * \\ * & 0 & 0 & * & * \end{pmatrix}$$

Figure 2. The operator system S_{C_5}.

Dr. Ivan Todorov is one of our newest additions to the faculty, joining us in Spring 2019. He obtained his PhD degree in 1999 from the University of Athens (Greece), in the area of Operator Algebras. He has held academic positions at the University of the Aegean (Greece) and Queen's University Belfast (UK), as well as visiting research positions at the Fields Institute for Research in Mathematical Sciences (Toronto) and the Isaac Newton Institute for Mathematical Sciences (Cambridge). Dr Todorov works in Functional Analysis and its interactions with Quantum Information Theory and Abstract Harmonic Analysis. His research is predominantly interdisciplinary and seeks a better understanding of some fundamental principles of Quantum Physics, such as entanglement, non-locality and no-signalling, using operator theoretic methods. He also employs the phenomena of operator transference to study questions about topological groups with operator algebraic methods. Major roles in his research are played by non-commutativity, positivity and order, and their multiple manifestations in operator algebra theory and quantum information.

Designing an Optimal Bus Route: A Collaboration with DART

PAK-WING FOK

From left to right: Pak-Wing Fok, Jared Kauffman (DART Fixed-Route Planner), David Dooley (DART Senior Planner), Hasan Mahdi (Mathematics and Economics), Miguel Fuentes (Mathematics, Computer Science, Political Science), Joy Kitson (Applied Mathematics, Computer Science) and Matthew Gargano (Mathematics, Computer Science)

MATH512 ("Contemporary Applications of Mathematics") is a capstone course for many of our math majors. One important difference about this course compared to traditional ones is that it encourages students to utilize the skills and knowledge accumulated from other classes to solve a real-world problem using mathematical thinking and modeling. There are no exams or traditional problem sets. Students spend the entire semester working on their project as a team. They give presentations in order to update their classmates and the instructor on their progress.

In Fall 2019, one of the student teams, consisting of **Miguel Fuentes**, **Hasan Mahdi**, **Joy Kitson** and **Matthew Gargano** worked on a problem that was posed by **Jared Kauffman** and **David Dooley**, two route planners who work at DART, the **Delaware Authority for Regional Transit**. While DART runs the bus routes in Delaware, historically the number of people who ride DART buses is very low. One possible reason for this is the sub-optimal placement of bus stops. For example, bus stops in a sparsely populated neighborhood may not be used regularly. However, the reasons for low usage of public transport are generally complex.

The problem faced by our students was to optimally place bus stops to maximize ridership while at the same time ensuring that the route time was not too large. To achieve this goal, DART planners provided 570 potential stops along Route 25, connecting Wilmington to Delaware City. The idea from our students was that ridership could be predicted at these potential stops using linear regression, K-nearest neighbours and/or random forest algorithms. The algorithms were trained on a dataset of currently existing stops, current ridership totals and census data.

By scoring any route through an objective function incorporating utilization and journey time (high utilization is rewarded while longer route times are penalized), our students designed a genetic algorithm to find high-scoring routes. They generated several possible optimal solutions, depending on the weights in the objective function. One of these routes is shown in Figure 1 (right). This route has a utilization of about 72, this is about 30% more than the current Route 25. Blue stops are mandated since they correspond to transit hubs while orange stops are on the current route. Red stops are new to the route.

At the end of the semester, the students gave a presentation to DART officials which stimulated a lively discussion. Messrs Dooley and Kauffman were enthusiastic about extending this work in a future project. This collaboration between The Department of Mathematical Sciences and DART is a great example of how mathematics can interface with industry while at the same time benefitting our students. Congratulations to our students for their achievements and for graduating last summer!

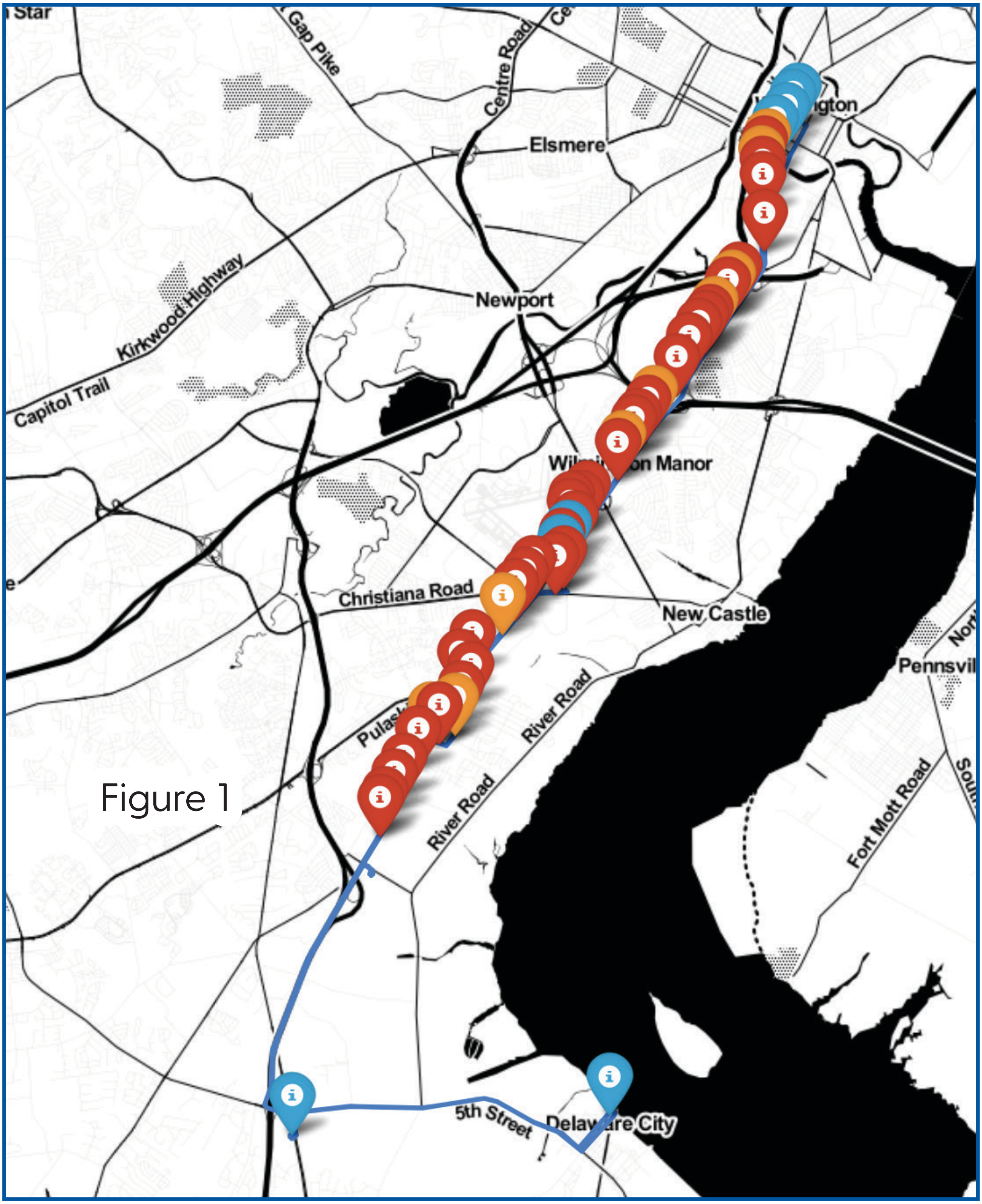


Figure 1. An optimal bus route 25 (connecting Wilmington to Delaware City) as predicted by a genetic algorithm. This distribution of stops is predicted to give 30% more ridership (utilization) that than the current route 25. Blue icons represent transit hubs at which the bus must stop. Orange icons are stops that are currently on the route. Red icons are new stops placed by the algorithm.

Sensitivity When Flipping Bits

SEBASTIAN M. CIOABA

On July 1, 2019, **Hao Huang**, an Assistant Professor at Emory University posted a 6 pages paper on arXiv titled *Induced subgraphs of hypercubes and a proof of the Sensitivity Conjecture*. Instantly, the paper created a huge buzz in the mathematics and computer science world [11–13]. Huang's paper has since appeared in the most prestigious journal in mathematics *Annals of Mathematics* [7]. In this short article, I describe his result, the motivation and the ideas of his beautiful proof. In October 2019, Hao Huang visited our Department, met with our students and faculty and gave an amazing talk describing his result. The same month, Fan Chung gave an interesting Colloquium talk in our Department describing Huang's results and other related problems.

The n-dimensional cube Q^n is the graph/network whose the nodes are all the binary words of length $n \geq 1$ over the alphabet {0, 1}, where two nodes are joined by an edge if they differ in exactly one position. It has 2^n vertices and each node has degree n, meaning it is adjacent to n other nodes. Figure 1 shows Q^n for $n \in \{1, 2, 3\}$. The main result of Huang's paper is the following theorem.

Theorem 1 (Huang 2019). *Let H be an induced subgraph with $2^{n-1} + 1$ vertices of the n-dimensional cube Q^n. Then the maximum degree of H is at least $\sqrt{n}$.*

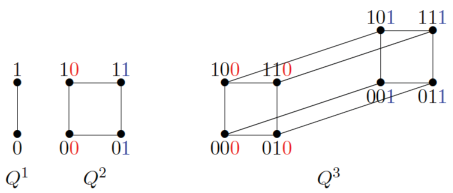


Figure 1: The n-dimensional cube for $n = 1, 2, 3$.

This says that no matter how one chooses more than half of the vertices in Q^n, there exists one vertex that is at adjacent to at least $\sqrt{n}$ vertices from our set. In Figure 1, no matter how one chooses 5 nodes of Q^3, one will be connected to at least $\lceil\sqrt{3}\rceil = 2$ others. While a case analysis may work for small n, such an approach would be hopeless for larger n as the number of possible cases grows exponentially in n. Huang's theorem is best possible in several ways. First, $2^{n-1} + 1$ cannot be replaced by anything smaller. The nodes of Q^n can be split into two groups of size 2^{n-1} each: the nodes of even support on one side (000, 011, 101, 110 in Q^3 in Figure 1) and the ones of odd support. There are no edges between nodes whose support has the same parity so one could choose up to 2^{n-1} nodes of even support before any edge appears. Secondly, Chung, Füredi, Graham and Seymour [3] showed in 1988 that there is always a way to choose $2^{n-1} + 1$ nodes such that $\Delta(H) = \sqrt{n}$ when n is a perfect square. These authors held the record on this problem ($\Delta(H) \geq \log_2 n$) before Huang.

I will briefly describe now how Theorem 1 is related to important problems in computer science. The alphabet of computers has two letters/bits: 0 and 1 and their language consists of words over this alphabet. In theoretical computer science, researchers study boolean functions which can be thought of as boxes where one inputs binary words (typically of the same length) of 0s and 1s and the output is 0 or 1. Think about a form you fill where the questions have only yes/no answers. Your responses form a string of 0s and 1s (0 for no and 1 for yes) and determine whether your form gets approved (output 1) or not (output 0). Another more philosophical example is thinking about a given period in one's life and how each binary decision determined a particular outcome. Remember that 1 + 1 = 0 for bits. Some examples of boolean functions are the two variable functions:

$$\mathrm{AND}(x_1, x_2) = x_1x_2$$

$$\mathrm{OR}(x_1, x_2) = x_1 + x_2 + x_1x_2$$

Given a boolean function $f: \{0, 1\}^n \to \{0, 1\}$ whose inputs are all the binary words of length n (for some natural number n) and a specific input $x = (x_1, \ldots, x_n)$ one can define the sensitivity $s(f, x)$ of the function f with respect to the input x as the number of entries j where $f(x)$ changes when we flip the bit x_j to $1-x_j$ and leave the other bits unchanged. For example, $s(\mathrm{AND}, 01) = 1$ as flipping the first bit in 01 changes the value of AND(01) while flipping the second bit does not. In other words, $0 = \mathrm{AND}(0, 1) = \mathrm{AND}(0, 0) \neq \mathrm{AND}(1, 1) = 1$. The sensitivity of the function f is now defined as $s(f) = \max_x s(f, x)$. For our examples, $s(\mathrm{AND}) = 2$ and $s(\mathrm{OR}) = 2$.

The binary words of length n can be interpreted as the nodes of the n-dimensional cube Q^n seen earlier. Note that the sensitivity of a function f with respect to a node $x \in Q^n$ is the number of neighbors y of x where $f(y) \neq f(x)$. The words w such that $f(w) = 1$ induce a subgraph H of the graph Q^n while the words z where $f(z) = 0$ induce the complementary subgraph $Q^2 \setminus H$. For the AND function defined above, H is the subgraph induced by 01, 10, and 11 while $Q^2 \setminus H$ is the just one vertex 00 (see Figure 1 for $n = 2$).

In computer science, a lot of research is devoted to understanding how various sensitivity measures relate to each other. The block sensitivity of a boolean function $f: Q^n \to \{0, 1\}$ with respect to a given input x is the maximum number of disjoint blocks $B_1, \ldots, B_k$ such that flipping the value of each of the entries whose index is in the set B_j changes the value of $f(x)$ for any $1 \leq j \leq k$. The block sensitivity $b(f)$ is the maximum of $b(f, x)$ over all possible $x \in Q^n$. As an example, consider the function $g: Q^8 \to \{0, 1\}$ whose output is 1 on words of length 8 having 4 or 5 ones and 0, otherwise. Flipping any of the first 5 bits in $x = 00000111$ will change the value of $g(x) = 0$ and this leads to $s(g) = 5$. For $y = 11110000$, flipping the bits in either one of the disjoint blocks {1}, {2}, {3}, {4}, {5, 6}, {7, 8} will change the value of $g(y) = 1$ giving $b(g) = 6$. It is not too hard to see that $s(f) \leq b(f)$ for any boolean function f and a natural question is how well can one bound the block sensitivity in terms of the sensitivity. This was posed as a conjecture in 1992 by Nisan and Szegedy [10] and became known as the Sensitivity Conjecture.

Conjecture 2 (Sensitivity Conjecture). There exists an absolute positive constant C such that for any boolean function f

$$b(f) \leq s(f)^C \tag{1}$$

Several authors (Rubinstein 1992, Virza 2011, Ambainis and Sun 2011) constructed boolean functions where the block sensitivity $b(f)$ is quadratic in the sensitivity $s(f)$ so if it exists, the constant C in the above conjecture must be at least 2. Kenyon and Kutin 2004 proved that for any boolean function $b(f) = O(e^{s(f)}\sqrt{s(f)})$.

It is known that any function $f: \{0, 1\}^n \to \{0, 1\}$ can be represented by a unique multilinear polynomial in n variables and we denote by deg(f) the degree of such polynomial. In 1992, Nisan and Szegedy proved that for any boolean function f,

$$b(f) \leq 2\deg(f)^2. \quad (2)$$

With (1) as a goal, a natural thing to do would be finding an upper bound for $\deg(f)$ in terms of the sensitivity $s(f)$. In 1992, Gotsman and Linial [6] obtained an equivalence between such a result and a problem involving the cube Q^n. For any induced subgraph H of Q^n, denote by $\Gamma(H) = \max(\Delta(H), \Delta(Q^n \setminus H))$, where $\Delta(H)$ is the maximum degree of H and $\Delta(Q^n \setminus H)$ is the maximum degree of $Q^n \setminus H$.

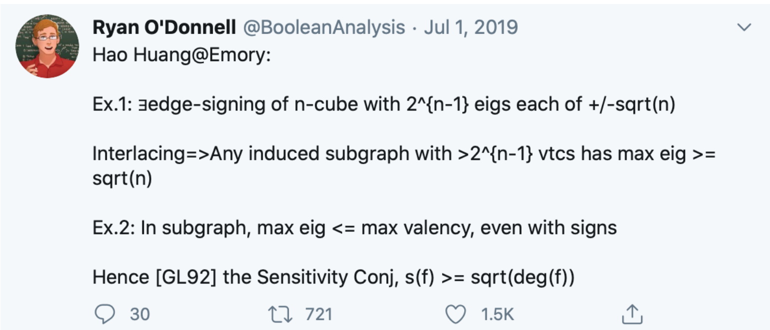
Theorem 3 (Gotsman-Linial 1992). *The following statements are equivalent:*

1. *For any boolean function* $f : Q^n \to \{0, 1\}$, $\sqrt{\deg(f)} \leq s(f)$.
2. *For any induced subgraph H of* Q^n *with* $2^{n-1} + 1$ *vertices or more,* $\sqrt{n} \leq \Gamma(H)$.

Gotsman and Linial conjectured that the above statements are true and that is in fact, what Hao Huang proved in Theorem 1. Combining his result with (2), one gets that

$$b(f) \leq 2s(f)^4, \quad (3)$$

proving the Sensitivity Conjecture with $C = 4$. There is still some work to do here as the best examples known so far have a quadratic gap between $b(f)$ and $s(f)$. For the remaining part of the article, I will describe Huang's proof of Theorem 1. This proof was summarized in a tweet by Ryan O'Donnell from Carnegie Mellon and I will briefly explain each point below.


Ryan O'Donnell @BooleanAnalysis · Jul 1, 2019
Hao Huang@Emory:

Ex.1: ∃edge-signing of n-cube with 2^{n-1} eigs each of +/-sqrt(n)

Interlacing=>Any induced subgraph with >2^{n-1} vtcs has max eig >= sqrt(n)

Ex.2: In subgraph, max eig <= max valency, even with signs

Hence [GL92] the Sensitivity Conj, s(f) >= sqrt(deg(f))

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The starting point of Huang's proof is a well know fact that Q^n can be constructed recursively as follows. For $n \geq 2$, take two disjoint copies of Q^{n-1}. The nodes in one copy get a 0 and the nodes in the other copy get a 1 appended to their label and then we add a perfect matching between these two copies (see Figure 1 for $n = 2, 3$).

The key insight of Huang's proof is constructing the following signed adjacency matrix of Q^n. This was also done recursively as follows:

$$A_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ and } A_n = \begin{bmatrix} A_{n-1} & I \\ I & -A_{n-1} \end{bmatrix},$$

for $n \geq 3$. It is easy to show by induction on n, that $A_n^2 = nI$. This means that A_n has two eigenvalues: $\sqrt{n}$ and $-\sqrt{n}$, each with multiplicity 2^{n-1}. Note that the usual adjacency matrix of Q^n has eigenvalues n-$2j$ of multiplicity $\binom{n}{j} - \binom{n}{j-1}$ for $0 \leq j \leq n$. Thus, using the signed adjacency matrix A_n, one *compresses* the specrtrum of the cube Q^n into two small eigenvalues $\sqrt{n}$ and $-\sqrt{n}$. One can show that $\sqrt{n}$ is the best you can do here [2].

Interlacing means Cauchy's eigenvalue interlacing for symmetric matrices. For a real and symmetric $n \times n$ matrix A with eigenvalues $\lambda_1(A) \geq \ldots \geq \lambda_n(A)$, there is some control for eigenvalues when deleting rows and columns.

Theorem 4 (Cauchy Interlacing). *Let A be an* $N \times N$ *symmetric real matrix. If B is a principal* $m \times m$ *submatrix of A (for some* $m < N$*), then the eigenvalues of B interlace the eigenvalues of A, meaning that* $\lambda_j(A) \geq \lambda_j(B) \geq \lambda_{j+N-m}(A)$, *for any* $1 \leq j \leq m$.

The way to tie all these things together goes as follows. Take an induced subgraph H of Q^n with $2^{n-1} + 1$ vertices. Using the signed adjacency matrix A_n constructed above the Cauchy interlacing (with $N = 2^n$, $m = 2^{n-1} + 1$, $j = 1$), we get $\lambda_1(H) \geq \lambda_{2^{n-1}}(A_n) = \sqrt{n}$. The last straw is another well known result, namely that the maximum degree of a graph always beats the largest eigenvalue so $\Delta(H) \geq \lambda_1(H)$. Hence, $\Delta(H) \geq \sqrt{n}$, qed.

The use of the signed ±1 adjacency matrices to represent graphs is not new. For example, see the 1960s work of Van Lint and Seidel [8] on equiangular lines and more recently the paper by Marcus, Spielman and Srivastava [9] showing the existence of bipartite regular Ramanujan graphs for any degree. Huang's proof solved a 30 year old conjecture and in less than one year, has stimulated a lot of research activity. Alon and Zheng [1] extended Huang's result and proved that it holds for a larger class of graphs. They also extended the ±1-signing of Q^n and introduced unitary complex signings of graphs where one can use ±1 and $\pm i$ as entries in an Hermitean adjacency matrix. Godsil, Levit and Silina [5] studied the connections with graph coverings and pointed out that the signed adjacency matrix of Q^n constructed by Huang, appeared in 1985 in a paper by Cohen and Tits [4] in a different context of finite geometries.

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Classroom Experiences from Six Universities Over Four Countries

NOVI BONG

I have had the opportunity to be involved with classroom experiences within mathematics departments at six universities over four different continents. I will share some of my observations of these experiences and how they compare to each other.

I began my study of mathematics at the **University of Indonesia**. I had to take a national entrance exam for public universities, competing with all students in Indonesia that were interested in Mathematics programs. There were about 60 new mathematics students accepted in my freshman year at the University of Indonesia. All of these students basically took classes together for the entire program. However, the class size did get smaller once students started their specialization. For the discussion sections, we maintained the same size of class as the lecture, which was not that effective. Both lectures and discussion section classes were 2 hours with no break in between. I found it was harder to follow the lecture during the second hour.

I was then lucky to have the opportunity to participate in a one semester student exchange at the **National University of Singapore** under an AUN-Network scholarship. In NUS, regular classes are conducted in a large lecture. I think it was more than 150 students in the class, but then they break down to smaller discussion groups. I also had the opportunity to attend an honors class. This class was small, about 12 students, and the material was much deeper than in the regular classes. The study environment in NUS was very competitive, and most of the class grades were assigned according to a bell curve.

I continued my master's degree under the Erasmus Mundus scholarship in the ALGANT program, where I could choose to study in two countries over the course of two years. I studied at the **Universiteit Leiden** in The Netherlands for my first year and then continued at UniversitèÑ Bordeaux 1 in France for my second year.

In the Netherlands, you could attend any class that you were interested in. There was no formal registration. One would be registered formally only if he/she took the final exam. The classes were conducted in a lecture format, 2 hours long, with a short break in the middle. We also had regular homework. Most of the exams were oral, where the difficulty of successive questions increased or decreased based on how well you responded to the previous question. This was a unique experience and I felt more nervous taking a one-on-one exam in front of the professor. I didn't remember if there was a fixed length of time for the exam, but I felt that I didn't have enough time to write down many of my calculations. Since questions were presented one at a time, there was no chance for me to browse the later questions and return to the earlier ones. You have 2 chances for the exam, so if you fail the first one, you can make another appointment to retake the exam. What I really liked during my year in the Netherlands was that they had a list of national mathematics courses. Due to the small size of the country, students were allowed to travel to other universities to take courses. There was a good opportunity to meet a lot of mathematics students all over The Netherlands. Even though I was a student in Leiden, I took courses in Universiteit van Amsterdam, Utrecht Universiteit etc. So, students had a wide variety of experiences. The cost of travel was covered by the Dutch government through reimbursement.

In **Bordeaux**, the classes were also in a lecture format; some of them in English and some of them in French. I had to take one class in French even though I had no knowledge of the language. There were no regular assignments. The exams were written and each was 3-4 hours long. It was a bit difficult to measure how well I was performing in the class because there was nothing graded until the midterm and final exam.

After finishing the Master's degree, I took a year off to work, and then I continued my PhD at the **University of Newcastle, Australia**. In Australia, the undergraduate program is for 3 years, but students have the option to continue to a fourth year, which involves further study in a particular discipline area. This one-year qualification taken after an undergraduate degree is called an Honours degree. After the Honours degree, students can continue directly to a PhD program without a Master's degree. There are no mandatory courses during the PhD program, so the start date is flexible. However, before applying to a PhD program, the candidate should have a supervisor in that university, so that research can be conducted as soon as a student is admitted.

Since the funding for the PhD program is a scholarship, being a teaching assistant is optional. Students will earn more money if they choose to be a TA (there we called them "Demonstrators"). I was encouraged by my supervisor to be a TA. In the discussion sections that I taught, we applied the active learning method. The professor provided the worksheet and the demonstrator handed the worksheet to the students and they were asked to work in a group, on the board. Students were given colorful markers, each member had different color, and each group occupied half of a board. I had to walk around the classes and ask each group to present their solution. If the solution is written in blue, I would ask students with different a color marker to explain the process, to make sure everybody in the group understood the solution. This is quite effective because the students got hands-on experience. This was also possible because the discussion section is once a week for 120 minutes. The last half an hour is used for a quiz.

Starting my first job in academia at the **University of Delaware**, I taught my first large class of 98 students. I like the idea of 50 or at most 75 minutes sessions of lectures/ recitations. However, it was tricky to try to apply active group discussions as I did in Australia, within 50 minutes, because it takes some time for students to get ready to work. On top of that, if we have to give a discussion quiz during the sections, that itself takes at least 20 minutes of the class time.

Student life is different in each country. In Indonesia, students can choose to live in the dorm, and the cost was around $10 per month when I was there. There were also a lot of private boarding houses within my campus. Meanwhile in Singapore, students have to compete to get in the dorm. Not only should students achieve good grades, but they also have to participate in extracurricular activities. The better the dorm, the more activity points they need to continue staying there. In Europe, they have private dorms that are linked to the university. Students can apply for a subsidy for rent from the government. It is interesting so see that most students live on campus in the US.

Comparing study habits, I would say that NUS has the most competitive students. In my dorm, seeing students studying at 7 AM everyday was very common. Their weekend was also spent in the library. It was very hard to find a spot in any study room, beginning several weeks before the exam. Students in Europe also have a good attitude in studying. They also have a strong, solid background in mathematics. From my observation, students at UD or Newcastle have a broader range of preparedness. Some students have a very good understanding of mathematics while others have to work on learning basic knowledge. However, students at these two universities are more active in asking questions. I am also very impressed to see how well students with disabilities are being accommodated here.

I am trying to adjust what I have experienced in different universities to come up with a strategy that works well here at UD. In my lectures, I will try to give couple of exercises (if time permits), so students have hands on experience. From my own experience, I felt that I understood when I just listened to the explanation of others but sometimes I was clueless when I had to attempt a problem myself. During my office hours, I apply the active learning technique where students have to write their attempts to a problem on the board and we discuss. I try my best not to give a solution, but instead, ask questions that will challenge their understanding to the concept. Eventually, they will have to work out their solutions.

My Time as a Math Major: Looking Back

GIN WANG

Jingibang (Gin) Wang *was a student in the Mathematics and Economics program at Delaware, graduating in 2016. His summer research experience was with Pak-Wing Fok. He went on to receive a Master of Science degree in Applied Analytics at Columbia University and then later joined Merrill Lynch, Pierce, Fenner & Smith Incorporated, an investment bank in New York City.*

I came to the United States by myself for college when I was 19. As a foreign national and non-native English speaker, college life wasn't easy at the beginning - living in a foreign country alone fresh out of high school, I had to overcome many challenges such as the language barrier and culture shock. I was blessed to have my adviser, Paul Sulzer, and the friends I met from my dorm floor who helped me with starting a completely new life. There were certain times I felt very depressed (e.g. feeling out-of-place culturally, encountering random discrimination, or struggling to balance work and fun). What I learned from those hard times was that:

1. You can't really change other people's perception of you, and what other people think really doesn't matter.

2. You have to do what you have to do and make the best out of it.

I'd call the two lessons I learned from the hardships of my freshmen year confidence and ambition. These two traits prepared me for my life and work the most. Then I started exploring different possibilities in UD - besides my academics (I graduated Cum Laude and was awarded the Carl Rees Scholarship in Mathematics), I had a part-time job as a teaching assistant for the computer science department, volunteered as a Chinese language tutor, joined a fraternity, co-founded a student activity club, owned a start-up business, worked as a summer researcher, studied a second foreign language, etc. I tried many things, and not only I found what I love, but these experiences also shaped who I am today. I was surprised to realize there were so many resources I could use at UD, and there were countless opportunities for me to explore and grow. UD is sometimes perceived as a party school, but my advice to new Blue Hens is this: life beyond parties and comfort zones is spectacular!

The research experience at the University of Delaware was simply incredible. As an undergrad student researcher, first things first, I learned many new things. It helped me realise which direction I would want to go: academia or industry? It was a fun academic experience, but its touch on real life scenarios got me really interested in industry problems. Second, I got to sharpen my mind tackling problems related to real life. I can't really speak on behalf of those who are in academics and research, but from an industry standpoint, and from my internship and work experience in investment banking and venture capital, it is important to have a "framework" to solve a problem or make a decision, otherwise you will be lost; the research experience definitely helped me shape my "framework". This framework, or thinking process helped me analyze more thoroughly and rationally during my internship and in the workplace, which made me stand out. And the research was the first time I got to manage a workflow independently - I learned new things, implemented and processed what I learned, and delivered what I learned. Working very independently is an ability gained from training and experience, not a talent. I have always loved building things, this research experience reinforced this – my mentor and I built something from scratch. This research experience also includes the relationship between a mentor and a mentee that I rarely found somewhere else. I hope that we will continue to stay friends.

Winter Research Symposium

PHILIPPE GUYENNE

The tenth annual **Winter Research Symposium** (WRS) of the Department of Mathematical Sciences was held on Friday February 14th 2020 in Gore Hall. **Lou Rossi**, the department's Chair, opened the symposium by noting the important contributions of our graduate students to this event. A big crowd was in attendance, including faculty, graduate students, family, friends and potential recruits for the coming year. About ten prospective students coming from as far as Utah were invited to visit the department and attend the WRS.

A number of departmental prizes were given at the start of the event. The winners of the AWM travel awards were announced by the president of the UD chapter, **Rayanne Luke**. **Amanda Mirzaei**, **Ben Nassau** and **Nick Russell** were each awarded travel reimbursements. **Lou Rossi** awarded the **Wenbo Li Scholarship Prize**, in memory of Dr. Wenbo Li. This cash prize recognizes an outstanding paper that is primarily authored by a graduate student. This year's winner was **Shukai Du** for his paper "New analytical tools for HDG in elasticity with applications to elastodynamics" which has been published in the journal *Mathematics of Computation.*

There were seven talks on a wide range of topics given by graduate students. Of these, six were senior students (**E. Bergman**, **H. Eruslu**, **M. Fuentes**, **N. Mirzaei**, **B. Nassau**, **J. Troy**) as well as the prize recipient (**S. Du**). There were also ten posters presented and displayed in Gore Hall's atrium, which together with the talks showcased the health and growth of our graduate program.

In addition, there was an invited alumnus talk. This year's speaker was **Bryan Petrak** who is currently an engineer working for Boeing in Oklahoma. He received his Ph.D. in mathematics from our department under the direction of **Gary Ebert** and **Felix Lazebnik** in 2012. His thesis topic was finite Figueroa planes. During his seven years with the Boeing company, he has worked as a software engineer, a systems engineer and currently as an electrical engineer. He has supported upgrades to military platforms including the B-1 and E-3 platforms. He currently works on the E-7 platform. Dr. Petrak spoke of the topic of multi-sensor data fusion, which involves sophisticated techniques from statistics, optimization and control theory.

DR. BRYAN PETRAK

At the end of the WRS, awards for best posters were given. The posters were judged by two faculty of the department using a detailed rubric. The two best poster awardees were **Rayanne Luke** and **Nick Russell**. This event made a good impression on the recruits. One recruit commented that he "had a great time. The camaraderie among the graduate students, and their love of being at Delaware, was easy to see". We would like to thank **Daytonia Campbell** as well as all the graduate students and faculty who helped make the WRS a successful event again this year.

Becoming a Data Scientist: Studying Data Science at UD

DOMINIQUE GUILLOT

As data analysis is becoming critical to more and more businesses, the demand for qualified data scientists has grown significantly in the past years. According to LinkedIn, hiring in data science has increased by 37% over the last three years, with many firms struggling to meet their recruiting objectives. This offers a great opportunity for those with skills in computer science, mathematics, and statistics who are interested to start a career in the industry.

The University of Delaware recently launched its new Master in Data Science degree, a collaboration between the Department of Mathematical Sciences, the Department of Applied Economics and Statistics, and the Department of Computer and Information Sciences. With more than 65 faculty members affiliated to the program, the degree offers a flexible set of core requirements to provide an interdisciplinary training. The program is aimed at providing a solid background in the methods behind data science so that graduates can be successful in the booming data science job market, and at the same time be better prepared for the next methods to come along.

The degree awarded to those who complete this program is a Master of Science with a major in Data Science. The degree is also offered in combination with a limited number of Bachelor degrees as a "4+1"' program. A total of 30 credits is required, with the option of writing a thesis. The program is now fully operating, with two students who graduated in May 2020. The number of students is expected to more than triple to over 40 students in the Fall 2020.

For more details, please visit **www.msds.udel.edu**

Department Hosts Math Modeling Camp

DAVID A. EDWARDS

From June 12–15, 2019, the Department hosted the 1**5th Annual Graduate Student Mathematical Modeling Camp**, which was organized by **Edwards and Rossi**. 23 students from 17 universities throughout the country converged on campus to learn how mathematics can be applied to real-world problems. Some of the students were already studying applied mathematics; for others, this was their first exposure to the discipline. The Department is particularly proud of the fact that this was the first camp to have majority-female participation.

After welcoming remarks from Drs. Edwards and Rossi, three invited faculty mentors presented problems to the entire group (outlined below) which are based upon real-world applications. The students then divided into teams to work with the help of the faculty mentors. Over the course of the next few days, the teams worked in an informal setting on basic mathematical modeling, analytical solutions, and numerical simulations.

Rotating through the rooms during the Camp, I was struck by the lively discussions on appropriate models, careful work on the tedious chore of nailing down parameter values (critical for any applied problem), and the mathematical progress made on numerical simulations. Students from the various teams reconnected at lunch or snack breaks. Discussions continued beyond the confines of campus at local pubs and restaurants, including at the opening night dinner at Klondike Kate's.

On Friday afternoon, the teams shifted from solving problems to preparing presentations and reports (which are posted at the Department's Web site). After closing remarks from Dr. Rossi on Saturday morning, a subset of each team gave a 30-minute presentation on the work they had done during the Camp. The presentations were quite professional, outlining the problem, the models, and the main results. All the faculty mentors were impressed with the caliber and amount of work the students accomplished.

The Camp was only half of the experience for the students, however. After the presentations were over, the students headed to the **New Jersey Institute of Technology** (NJIT) to attend the **Mathematical Problems in Industry Workshop** (MPI). At MPI, the students joined teams of faculty and postdocs to work on cutting-edge, research-level problems brought by industrial representatives. Buoyed by their experience at the Camp, the graduate students became valued members of the MPI teams, providing critical support to their modeling and computational endeavors.

The combined Camp-MPI experience is unique among US institutions, and gives mathematics graduate students experience in how to apply their knowledge to real-world problems, as well as exposure to companies who are looking to hire industrial mathematicians. As UD participant **Jerome Troy** put it, "The program ties together the methods we learned in class with how they can be applied to real-world problems. Working alongside students from other universities is a great way to learn new tools, and network with the faculty and camp mentors." Fellow UD participant **Rayanne Luke** added, "I had the opportunity not only to learn something while working with new collaborators but to lend my own bit of expertise to the problem. I would recommend the camp to students at any point in their graduate work, as I found it a worthwhile experience to practice my communication, modeling, coding, and writing skills."

Though the 2020 Camp had to be cancelled due to the coronavirus pandemic, the Department looks forward to hosting this exciting event again in 2021.

Here are the problems presented at this year's camp:

Dr. Daniel Anderson of George Mason presented a problem on "Drug Delivery on Contact Lenses". Many people have trouble administering medication with eye drops. They blink, the drops leak out of the eye, etc. Hence researchers are examining using contact lenses as an alternative means for sustained drug delivery for the treatment of eye disorders. To design these lenses effectively, researchers need mathematical models of the diffusion of chemicals both through the lens and in the tear film. The team aimed to develop predictive models for drug delivery in contact lens and tear films. **Dr. Braun**, who does research in tear films as it applies to the treatment of dry eye, dropped in during the Camp to provide scientific and moral support.

During the Camp, the team used the diffusion equation to solve for the drug concentration in the lens. This was then coupled to boundary conditions representing the effect of the tear film both ahead of and behind the lens. They then considered the effect of blinking to "pump" the fluid in the tear film. After introducing suitable scalings and parameter values, the team was able to generate numerical simulations of the drug concentration throughout the eye.

Dr. Richard Moore of NJIT presented a problem on "Cutting with Water: From Fish to Fracking." Abrasive water jets (with or without grit) are used as precision cutting tools in many industries. However, at higher speeds, the cuts can manifest undesirable striations, blank regions, or bevels. The team worked to develop a basic understanding of the operation of the abrasive water jet and the mechanisms underlying the problems described above.

During the Camp, the team used ordinary differential equations to model the volume removed from the cutting surface by the jet, how the jet's velocity decreases as it penetrates into the material, and how the grit in the jet decays with depth of cut. They were then able to model numerically how the jet's oscillations leads to striations in the cut surface.

Dr. Pejman Sanaei of New York University (himself a former Camper) presented a problem on "Flow and Fouling in Elastic Membrane Filters." Most models of membrane filters assume the pores are cylindrical tubes of fixed radius. Real membranes have interconnected pores that may branch or expand as fouling occurs. The team worked to understand these more complicated models, and how fouling occurs under a constant pressure drop and constant-flux scenario.

During the Camp, the team treated the pores as both thick- and thin-walled cylinders, calculating material quantities using partial differential equations. By treating the walls as elastic, the group was able to compute how the radius of the cylinders would change as the pores became blocked. This then led to a numerical computation of the number of unblocked cylinders (a measure of filter performance) over time.

Graduate Students Help with Delaware Science Olympiad

NICK RUSSELL

"How many glasses of milk would fill the Chesapeake Bay?"

"Assuming the population density of Delaware, how many people could live on Mars?"

"If a 160-pound human is running at constant speed and consumes a birthday cake every hour, then how many miles per hour must they run in order to burn the resulting calories?"

In the fall of 2017, the **Delaware Science Olympiad** (DSO) reached out to the Department in order to ask for some assistance on an event for the 2018 DSO. The DSO is the state run competition of the **National Science Olympiad**, which hosts competitions for middle school (ages 11-15) and high school (ages 15-18) students with a myriad of ever-changing STEM events drawing from problems in biology, chemistry, physics, and mathematics. The DSO has taken place annually in March since 1985, where 50-60 teams compete for a chance to go on to the National Tournament in late spring. The DSO runs on the volunteers, who have various backgrounds in STEM and non-STEM professions, that help construct and adjudicate all 21 events while providing a safe and positive atmosphere for young students to grow their love for science and mathematics.

The event that the math department first worked on was called Fermi Questions. This event requires students to quickly estimate seemingly impossible questions, like the ones above, by utilizing dimensional analysis and making reasonable assumptions to approximate answers, which are given in powers of 10. As an example, if a student says that there would be about 20 million glasses of milk which would fill the Chesapeake Bay, since 20 million can be written as 2 x 10^7, the student would write down 7 as their answer. The closer they are to the correct magnitude, the more points they get. Each team, which consists of 2-3 students, have 50 minutes to answer 25 of these questions.

After receiving this email from the DSO, the faculty from the department forwarded the information to the graduate student-led SIAM chapter and asked if they would like to be involved with this event. The SIAM board, lead by president **Samuel Cogar**, took on the challenge of constructing and administering the Fermi Questions exam on the day of the high school event, along with hosting a workshop in early February to help students prepare for this event which last presented itself in 2013 (the answers to the questions above, which appeared on the exam, are at the end of the article). The event inspired us all, as we realized how much of an impact this whole tournament had on the students. This was a non-judgemental and welcoming place where students could foster their love for science and work with their friends and teammates to build, experiment, and critically think. Because of the impact it had on us, I decided to join the Board of Directors of the DSO as the Chair of the Inquiry and Nature of Science events.

For the 2019 tournament, the UD Department of Mathematical Sciences expanded its role by being the event supervisors for 4 events, spanning both middle and high school competitions: Fermi Questions, Geologic Mapping, Code Busters, and Game On. Geologic Mapping tested students on constructing, utilizing, and analyzing topographic and cross section maps; Codebusters had teams cryptanalyze and decode encrypted messages using cryptanalysis techniques for historical and modern advanced ciphers; and Game On had students design and build a video game using the program Scratch incorporating the science theme provided to them. We had 18 graduate student volunteers that helped with every facet of this process, including holding a Saturday workshop for students on their respective events. For the 2020 tournament, even though the middle school tournament was cancelled due to COVID, 17 graduate students ran two events for the high school tournament, as well as 5 undergraduates from the Cybersecurity Scholars program here are the UD which helped design the event and workshop for the Codebusters competition.

The exuberant smiles, the joy on the students faces when they solved a problem, and their passion for science were so powerful, and we have drawn off that enthusiasm. The care, intensity, and devotion that the graduate students took with this process spoke volumes about how much community outreach means to them. It is vital for people to see what mathematicians are really like and to influence the next generation to do something great. Even though the Science Olympiad is heading into some uncertainty over the next year, the graduate students are here and eager to help grow the DSO's reach, one Fermi question at a time.

If you would like to volunteer for any part of the Delaware Science Olympiad, please reach out to **nrussell@udel.edu** or **DEScienceO@gmail.com**.

Answers: 14, 10, 1

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Pak-Wing Fok

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Sebastian Cioaba

David Edwards

Pak-Wing Fok

Dominique Guillot

Philippe Guyenne

Louis Rossi

Nick Russell

Ivan Todorov

Gin Wang

DESIGN

Lukas Emory

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Department of Mathematical Sciences
University of Delaware
Ewing Hall
Newark, DE 19716

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