FIRST TIER

Linear algebra

Topics covered on the exam include the following. Most of these topics are discussed in Math 672.

1. Subspaces, bases and dimension (Chapters 1 - 2, [1]).
2. Linear transformations and matrix representations (Chapter 2, [1]; Chapters III - IV, [3]).
3. Determinants and rank (Chapter VI, Section V.3, [3]).
4. Inner products and inner product spaces (Chapter 6, [1]).
5. Linear functionals, adjoints, and dual spaces (Chapter 6, [1]).
6. Bilinear forms, Hermitian forms, and quadratic forms (Chapter V, [1]; Sections IV.A - IV.C, [2]).
7. Eigenvalues, eigenvectors, and characteristic polynomials (Sections VIII.1 - VIII.2, [3]).
8. Cayley-Hamilton Theorem (Sections III.A - III.C, [2]).
9. Operators on inner product spaces and Spectral Theorems (Chapter 7, [1]; Section VIII.3 - VIII.6, [3]; Section III.D, [2]).
10. Jordan Canonical Form (Chapter XI, [3]; Section III.E, [2]; Chapter 8, [1])

Chapter and section numbers refer to


Real analysis

Topics covered on the exam include the following. Except the part on multivariable calculus, most of these topics are discussed in Math 600.

1. Metric Spaces: open and closed sets, compactness, connected sets, complete sets, continuous functions on metric spaces ([1], Chapters 3 and 4, [3] Chapter 2).
2. Continuity and Differentiation: mean value theorem, Rolle's theorem, Taylor's formula, derivatives of vector valued functions, uniform continuity, monotonic functions ([1], Chapters 5 and 6, [3] Chapter 4).
3. Infinite Sequences and Series: Limit superior and limit inferior, monotonic sequences, alternating series, absolute and conditional convergence, power series, tests for convergence of series, rearrangement of series ([1], Chapter 8, [3] Chapter 3).


6. Functions of Several Variables: Directional derivatives, the total derivative, Jacobians, inverse function theorem, implicit function theorem, extrema problems ([1], Chapters 12 and 13).

7. Vector Calculus: Line integrals, Green's theorem, surface integrals, Stokes theorem, the divergence theorem ([2], Chapters 10, 11 and 12).

Chapter numbers refer to


[SYLLABI FOR SECOND TIER EXAMS START IN THE NEXT PAGE]
SECOND TIER

Integration and complex variables

Topics covered on the exam include the following. Most of these topics are discussed in Math 602.

1. Construction and properties of the Lebesgue measure
2. Lebesgue measurable and integrable functions
3. The dominated convergence theorem, Fatou’s lemma, the monotone convergence theorem, and the bounded convergence theorem
4. Analytic functions, Taylor series, Cauchy’s theorem, the generalized Cauchy integral formula
5. The maximum modulus principle and Liouville’s theorem
6. Laurent series, the residue theorem and applications to computation of integrals

References are:

- [1], Chapter 10 and [2] Chapters 2-4 and 7 for items 1-3.


Applied mathematics

Topics covered on the exam include the following. Most of these topics are discussed in Math 617.

1. Ordinary Differential Equations
   a. Fourier series (use in PDEs) [1] ch. 3.
   b. Sturm-Liouville theory (as preparation for eigenfunction expansions of PDEs) [1] ch. 5.
   c. Green’s functions [2] §5.5
   d. Bessel and Legendre functions (as eigenfunctions for PDEs in alternate geometries) [1] ch. 7.
2. Second-Order Linear PDEs
   a. Separation of variables [1], ch. 2
   b. Fourier and Laplace transform methods [1] ch. 10 and 13
   c. The diffusion equation (maximum principle) [1] ch. 2
   d. Laplace’s equation (maximum principle, Poisson’s integral formula): [1] §9.5
   e. The wave equation (characteristics, d’Alembert’s solution): [1] §12.3

Section and chapter references are given from

Numerical methods

Topics covered on the exam include the following. Most of these topics are discussed in Math 611.

1. Polynomial interpolation:
   a. Lagrange and Newton form [QSS, 8.1,8.2].
   b. Piecewise interpolation in 1 and 2D [QSS, 8.3,8.5].
   c. Splines [8.6,8.7].
2. Numerical Integration and Finite Differences:
   b. Singular integrals [9.8].
   c. Orthogonal polynomials [10.1].
   d. Gaussian quadrature [10.2, 10.4].
   e. Approximation of derivatives [10.10]
3. Numerical solution of Ordinary Differential Equations:
   a. Gronwall Lemma [11.1],
   b. One step methods [11.2],
   c. Stability and consistency [11.3]
   d. Difference equations [11.4]
   e. Multistep methods [11.5]
   g. Runge Kutta methods [11.8]
   h. Stiff Problems [11.10]
4. Finite Differences
   a. Finite difference approximation to two point boundary value problems [QSS12.2]
   b. Discretization of the heat equation [13.2]

The section numbers refer to Numerical Mathematics by A. Quarteroni, R. Sacco, and F. Saleri.

Algebra

Topics covered on the exam include the following. Most of these topics are discussed in Math 650.

1. Elementary Group Theory
   a. basics (group, subgroup, cosets, Theorem of Lagrange) (Ch. 1,2)
   b. Homomorphisms, normal subgroups and Isomorphism Theorems. (Ch. 3)
   c. Classification of Finite Abelian Groups. (Ch. 7AB)
2. Commutative Rings
   a. Polynomial rings, PIDs and UFDs. (Ch. 16)
   b. Fields and field extensions, splitting fields. (Ch. 17)
3. Finite Fields (Ch. 21AB)
   a. Structure and uniqueness. Subfields. (Ch. 21A)
   b. Irreducible polynomials (and counting them). (Ch. 21B)

Chapter numbers refer to I.M. Isaacs, Algebra, A graduate course.

Stochastic processes

Topics covered on the exam include the following. Most of these topics are discussed in Math 631.
1. Markov chains (discrete-time) (Chapter 1)
2. Continuous-time Markov chain (Chapter 4)
3. Martingales (discrete-time) (Chapter 5)
4. Key examples:
   a. Random walks
   b. Birth and death processes
   c. Branching processes
   d. Markov chain Monte Carlo

Chapter numbers refer to R. Durrett, *Essentials of Stochastic Processes*