Centrality Measures In The Real World

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Given a record of all phone calls made in one town over a period of one month, who is the most influential person?

11 games into the college football season, if 13 teams are all 10-1, how do we (fairly*) decide which two teams play for the national championship?

If I want to advertise through Twitter by spending money to get 3 famous people to tweet about my product, how do I decide which people to pursue?

If I’m inventing a search engine, how do I decide which pages to rank the highest?
A real-world problem

- Suppose we are told that drug lords are using their phones to facilitate runs in a certain town, and we have all their phone records.
- How do we find the head people in this operation just by phone records?
Ideas?

- Find who makes the most calls. High volume = high importance.
Ideas?

- Find who makes the most calls. High volume=high importance.
- Most of the high-volume people are middle-men, taking orders from bosses and finding customers.
- The other high-volume callers are nicknamed “pizza shops” (incoming) and “fundraisers” (outgoing). They are almost always unimportant for networking.
Other Ideas?

- Find who only has incoming calls?
- Find who only has outgoing calls?
- Find large ‘cliques’ who all call each other but very few others?

Remember this problem...
Measuring Influence via Twitter

- How do we find out which tweeters are most influential?
Ideas?

- Whoever has the most tweets @ them is the most influential.
- Whoever has the most followers is the most influential.
- Whoever gets the most replies is the most influential.
Ideas?

- Whoever has the most tweets @ them is the most influential.
- Whoever has the most followers is the most influential.
- Whoever gets the most replies is the most influential.
- In 2012, Justin Beiber was top in all 3 categories, more than double anyone else, every single day of the year.
Even Worse

- In 2012, the closest that Justin Beiber ever came to not being number one was on April 18, 2012.

- On that day, Dick Clark was number 2. Dick Clark received 47% of the directed tweets that Justin Beiber did on April 18th.
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- In 2012, the closest that Justin Beiber ever came to not being number one was on April 18, 2012.

- On that day, Dick Clark was number 2. Dick Clark received 47% of the directed tweets that Justin Beiber did on April 18th.

- April 18, 2012 was the day Dick Clark passed away.
A Smarter Algorithm

- If A makes a call to B, and B makes a call to C, but A never called C, then B is on the shortest path between A and C.
- Idea: More shorter paths = more importance.

Kevin Bacon showed us that most shortest paths are length 6 or less.
Betweenness Centrality

- For all pairs of vertices in a graph, find the shortest path between them.
- Each occurrence of a vertex on a shortest path is one point.
- Total points (or a ratio therein) is the betweenness centrality.
- Idea: More points = more important.
BetweennessCentrality (jung 2.0 API)
jung.sourceforge.net/doc/apidocs/.../BetweennessCentrality.html - JUNG - java.lang.Object extended by edu.uci.ics.jung.algorithms.utilITIVEProcess... Computes betweenness centrality for each vertex and edge in the graph.

BetweennessCentrality java
logic.cs.unt.edu/taura/teaching/.../BetweennessCentrality.java - package edu.uci.ics.jung.algorithms.importance; import java.util... vertex * and edge has a UserData element of type MutableDouble whose key is 'centrality'.

Java/JBLAS: Calculating eigenvector centrality of an...
www.markhneedham.com/...java-jblas-calculating-eigenvector-centrality... Aug 5, 2013 - I recently came across a very interesting post by Kieran Healy where he runs through a bunch of graph algorithms to see whether he can detect...

calculating degree centrality in Java - Stack Overflow
stackoverflow.com/questions/.../calculating-degree-centrality-in-java - Sep 5, 2014 - I am having a problem where I need to calculate the degree centrality... The Java network libs for SNA analysis are a bit limited in my experience.

Java/JBLAS: Calculating Eigenvector Centrality of an...
java.dzone.com/articles/java-jblas-calculating - Aug 7, 2013 - The first algorithm he looked at was betweenness centrality which I've looked at previously and is used to determine the load and importance of...

BetweennessCentrality - Niraj - Sites - Google
https://sites.google.com/site/nirajjweb/home/.../betweenness-centrality - I use Java universal network graph library (JUNG) to calculate the betweenness centrality of nodes and edges. For this, I consider the following network structure...

gs-algo/ClosenessCentrality.java at master · graphstream/gs ...
https://github.com/graphstream/gs-algo/.../ClosenessCentrality.java - gs-algo/org/graphstream/algorithms/measures/ClosenessCentrality.java. Fetching... Constructor allowing to configure centrality attribute. Same as calling.

LinkedData-QA/Centrality.java at master · cgqueret/... - GitHub
https://github.com/cgqueret/LinkedData-QA/blob/master/.../Centrality.java - Playing around analysis of Linked Data. Contribute to LinkedData-QA development by creating an account on GitHub.

BetweennessCentrality (The GraphStream 1.2 API)
graphstream-project.org/apolis-algo/.../BetweennessCentrality.html - org.graphstream.algorithms. Class BetweennessCentrality. java.lang.Object extended by... Compute the "betweenness" centrality of each vertex of a given graph.
centrality - UCLA.edu
www.sscnet.ucla.edu/~cent-ans.ht... University of California, Los Angeles
To calculate betweenness centrality, you take every pair of the network and count how many times a node can interrupt the shortest paths (geodesic distance) between the two nodes of the pair. For standardization, I note that the denominator is \((n-1)(n-2)/2\). For this network, \((7-1)(7-2)/2 = 15\).

Centrality - Wikipedia, the free encyclopedia
Betweenness is a centrality measure of a vertex within a graph (there is also edge betweenness, which is not discussed here). Betweenness centrality quantifies the number of times a node acts as a bridge along the shortest path between two other nodes.

Betweenness centrality - Wikipedia, the free encyclopedia
Note that the betweenness centrality of a node scales with the number of pairs of nodes as implied by the summation indices. Therefore the calculation may be ....

Betweenness centrality - NetworkAnalyzer Help
med.bioc.mpi-inf.mpg.de/networkAnalyzer/help/2.7/ - Max Planck Society
The stress centrality [4, 14] of a node \(n\) is the number of shortest paths the fast algorithm by Brandes [4] for the computation of node betweenness centrality.

Centrality
www.sscnet.ucla.edu/~moody77/sb884/...class_centrality.... - Duke University
Conceptually, centrality is fairly straightforward: we want to identify which nodes ... UCINET, SPAN, PAJEK and most other network software will calculate these ....

Network Centrality
cs.byrdmaw.edu/Courses/cs380/spring2013/.../slide05_Centrality.pdf -
In each of the following networks, \(X\) has higher centrality than \(Y\) according to ... Freeman's general formula for centralization (can use other metrics, e.g.

Graph Processing: Calculating betweenness centrality for ...
www.markneedham.com/.../graph-processing-calculating-betweenness.... -
Jul 19, 2013 - Since I now spend most of my time surrounded by graphs I thought it'd be interesting to learn a bit more about graph processing, a topic my ....

Closeness centrality in networks with disconnected ...
toreopsahl.com/.../closeness-centrality-in-networks-with-disconnected-co... -
Mar 20, 2010 - A key node centrality measure in networks is closeness centrality ... The distance calculation in a directed network generally assumes that ...
Great Idea?

- Important people are on more shortest paths, especially celebrities and mavens.
- Pizza shops and fundraisers don’t call both ways, so they are not on shortest paths.
- Computation complexity is not much worse than quadratic, this is acceptable for a lot of problems.
- For several years, this was the preferred method by our government customer.
Airports

- We put every airport and flight in North America into a database and measured betweenness centrality.
- Houston being on the shortest path from Baltimore to San Diego means one point for Houston. Ties split the point.
- It turns out there was a landslide winner for “most important airport in the US.” More than double second place, triple third place.
## Betweenness Centrality

<table>
<thead>
<tr>
<th>Rank</th>
<th>Airport Name</th>
<th>Location</th>
<th>Betweenness Centrality</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ANC (Anchorage, AK, USA)</td>
<td></td>
<td>465272</td>
</tr>
<tr>
<td>2</td>
<td>FAI (Fairbanks, AK, USA)</td>
<td></td>
<td>215503</td>
</tr>
<tr>
<td>3</td>
<td>YYZ (Toronto, Canada, Canada)</td>
<td></td>
<td>131562</td>
</tr>
<tr>
<td>4</td>
<td>LAX (Los Angeles, CA, USA)</td>
<td></td>
<td>129246</td>
</tr>
<tr>
<td>5</td>
<td>SEA (Seattle/Tacoma, WA, USA)</td>
<td></td>
<td>125151</td>
</tr>
<tr>
<td>6</td>
<td>JFK (New York, NY, USA)</td>
<td></td>
<td>124927</td>
</tr>
<tr>
<td>7</td>
<td>HPN (White Plains, NY, USA)</td>
<td></td>
<td>121096</td>
</tr>
<tr>
<td>8</td>
<td>MIA (Miami, FL, USA)</td>
<td></td>
<td>120643</td>
</tr>
<tr>
<td>9</td>
<td>DEN (Denver, CO, USA)</td>
<td></td>
<td>120342</td>
</tr>
<tr>
<td>10</td>
<td>MSP (Minneapolis, MN, USA)</td>
<td></td>
<td>111188</td>
</tr>
</tbody>
</table>
Alaska has hundreds of airports. All of them fly through Anchorage or Fairbanks, mostly Anchorage.

Betweenness centrality counted each of them equal to New York City, even with only one flight per month.
Other Problems

- Justin Beiber obviously was first place in betweenness centrality, but almost all of his tweets were from “teenage girls.”
- College Football had one year with a “rock-paper-scissors” situation, 3 teams with similar schedules all beat each other for their only loss. They tie under this method.
New Method - An Example

\[ A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \]

\[ B = A + I = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \]
Eigenvalues

- Given a matrix $M$, an eigenvalue $e$ and eigenvector $v$ are such that $Mv = ev$. Further, for any exponent $n$, $M^n v = e^n v$.

- A “nice” matrix can be completely described by its eigenvalues and eigenvectors.
Eigenvalues

- Given a matrix $M$, an eigenvalue $e$ and eigenvector $v$ are such that $Mv = ev$. Further, for any exponent $n$, $M^nv = e^nv$.

- A “nice” matrix can be completely described by its eigenvalues and eigenvectors.

- Claim 1: The question relevant to probability is “What happens when the matrix is raised to an infinite exponent?”
Eigenvalues

- Given a matrix $M$, an eigenvalue $e$ and eigenvector $v$ are such that $Mv = ev$. Further, for any exponent $n$, $M^nv = e^nv$.

- A “nice” matrix can be completely described by its eigenvalues and eigenvectors.

- Claim 1: The question relevant to probability is “What happens when the matrix is raised to an infinite exponent?”

- Claim 2: When looking at large/infinite exponents of a matrix, all that really matters is the principal eigenvalue.
An Example

- In our example, the principal eigenvalue is 2.70559.
- The other eigenvalues are -1.851, -1.350, 1.056, -0.560, and 0.

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 & 0 \\
\end{bmatrix}
\]

\[
\mathbf{v}_0 = \begin{bmatrix}
0.092 \\
0.249 \\
0.581 \\
0.405 \\
0.514 \\
0.405 \\
\end{bmatrix}
\]
\[ d = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \]

\[ B = A + I = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 1 & 1
\end{bmatrix} \]

\[ Bd = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad B^2d = \begin{bmatrix} 2 \\ 3 \\ 2 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad B^3d = \begin{bmatrix} 5 \\ 7 \\ 8 \\ 5 \\ 4 \end{bmatrix} \]
\[ d_k = \frac{B^k d}{|B^k d|} \]

\[ d_1 = \begin{bmatrix} 0.577 \\ 0.577 \\ 0.577 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad d_2 = \begin{bmatrix} 0.447 \\ 0.671 \\ 0.447 \\ 0.224 \\ 0.224 \\ 0.224 \end{bmatrix}, \quad d_3 = \begin{bmatrix} 0.358 \\ 0.501 \\ 0.573 \\ 0.286 \\ 0.358 \\ 0.286 \end{bmatrix} \]

\[ B = A + I = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \]
\[ \mathbf{d} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]

\[ B = A + I = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \]

\[ \mathbf{d}_1 = \begin{bmatrix} 0.577 \\ 0.577 \\ 0.577 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{d}_2 = \begin{bmatrix} 0.447 \\ 0.671 \\ 0.447 \\ 0.224 \\ 0.224 \\ 0.224 \end{bmatrix}, \quad \mathbf{d}_3 = \begin{bmatrix} 0.358 \\ 0.501 \\ 0.573 \\ 0.286 \\ 0.358 \\ 0.286 \end{bmatrix}, \quad \mathbf{d}_{30} = \begin{bmatrix} 0.092 \\ 0.249 \\ 0.581 \\ 0.405 \\ 0.514 \\ 0.405 \end{bmatrix} \]
\[
d = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad B = A + I = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}
\]

\[
d_1 = \begin{bmatrix} 0.577 \\ 0.577 \\ 0.577 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad d_2 = \begin{bmatrix} 0.447 \\ 0.671 \\ 0.447 \\ 0.224 \\ 0.224 \\ 0.224 \end{bmatrix}, \quad d_3 = \begin{bmatrix} 0.358 \\ 0.501 \\ 0.573 \\ 0.286 \\ 0.358 \\ 0.286 \end{bmatrix}, \quad d_{30} = \begin{bmatrix} 0.092 \\ 0.249 \\ 0.581 \\ 0.405 \\ 0.514 \\ 0.405 \end{bmatrix}, \quad v_0 = \begin{bmatrix} 0.092 \\ 0.249 \\ 0.581 \\ 0.405 \\ 0.514 \\ 0.405 \end{bmatrix}
\]
Perron-Frobenius Theorem:
If M is a “nice” matrix then M has a principal eigenvalue such that all entries in its corresponding eigenvector are positive.

Gould’s Index:
The principal eigenvector, once normalized, gives an accessibility rank to each vertex.
Eigenvector Centrality

- The eigenvector centrality is defined by the normalized principal eigenvector, a.k.a. Gould’s Index, which has all positive entries.

- The principal eigenvector can be computed relatively easily with a power iteration method.

- In practice, the power iteration method is accomplished with edge lists, and can be multiplied in parallel.
The maximum centrality belongs to vertex 3, followed by 5.

Vertex 2 is 5\textsuperscript{th} highest, only exceeding that of 1.

Note that betweenness centrality of vertex 2 was second highest.
Google PageRank

- Google does not publicize their exact algorithm, but it initially used a variant of eigenvalue centrality.

- It is believed that the original PageRank randomly crawled the web, following random links on each page, and jumped to another page with probability ~10%.

- Pages visited more often have higher scores. Higher scores mean higher on search results.
Advantages

- Unimportant nodes, like Alaskan Airports and teenage girls, do not count for much each.
- Only having high out-degree does not boost the score.
- No human input is needed. This means no college football voting, no celebrity lists, no lists of suspected drug lords.
- Edge weights are allowed in the computation. This is huge for airports, and often for calls as well.
### New Results

<table>
<thead>
<tr>
<th>Code</th>
<th>City</th>
<th>State</th>
<th>Score</th>
</tr>
</thead>
<tbody>
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<td>ATL</td>
<td>Atlanta</td>
<td>Georgia</td>
<td>0.427686</td>
</tr>
<tr>
<td>ORD</td>
<td>Chicago</td>
<td>Illinois</td>
<td>0.248052</td>
</tr>
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<td>LAX</td>
<td>Los Angeles</td>
<td>California</td>
<td>0.228303</td>
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<td>DEN</td>
<td>Denver</td>
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<td>California</td>
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<td>BWI</td>
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New Results - Worldwide

<table>
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<tr>
<th>Code</th>
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</thead>
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<td>China</td>
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<td>PHL</td>
<td>Philadelphia</td>
<td>United States</td>
<td>0.105972</td>
</tr>
</tbody>
</table>
New Results

- College football uses a version of this algorithm, but it still incorporates voting. While they do not make their methods public, it is believed that voting weights the random steps when they are taken.

- Google still uses a later version of PageRank, eventually they shifted to make physical location a huge part of the score.
Twitter

* The most influential tweeter ...
Twitter

- The most influential tweeter ... is still Justin Beiber. A lot of celebrities do tweet to him, albeit not always positive.
- The insight we provided is that teenage girls make up a majority on twitter. Other groups definitely use it, but in the minority.
- Analytics provide huge insights into our data. We don’t always love those insights.
Twitter-like Data?

- The exact same algorithm was conducted on 2 data sets, one email and one telephony data.
- These data sets are far from complete, which made well-known clustering algorithms impossible to implement without restricting to a much smaller subgraph.
- Eigenvector centrality provided lists of possible seeds which led to much different starting groups, and many reports of much higher effectiveness on clustering algorithms.
Did We Find the Drug Bosses?

Bosses

Middle-men

Middle-men

Middle-men
Thank you very much!

Questions?