A class of regular functions related to univalent functions
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If \( g(z) \) is regular in the open unit disk \( E \), normalized by \( g(0) = 0 \) and \( g'(0) = 1 \), and there is a complex number \( \epsilon \), \( |\epsilon| = 1 \), such that \( \text{Re} \left[ \epsilon z g'(z)/g(z) \right] > 0 \) for \( z \) in \( E \); then \( g(z) \) is said to be a spiral function. The spiral functions are used to define a new class of regular functions: \( f(z) \) is in \( H \) if and only if \( f(z) \) is regular in \( E \), satisfies \( f(0) = 0 \) and \( f'(0) = 1 \), there exists a spiral function \( g(z) \) and a complex number \( \xi \) such that \( \text{Re} \left[ \xi z f'(z)/g(z) \right] > 0 \), \( z \) in \( E \). The class \( H \) contains the spiral functions and the close-to-convex functions; it also contains functions which are not univalent.

The dissertation has four chapters. The first chapter is concerned with preliminary definitions and the decomposition of \( H \) into subclasses.

In chapter two the methods of interior variation are used to solve two general extremal problems over \( H \).

Let \( f(z) = z + a_2 z^2 + \ldots \) belong to \( H \). Chapter 3 deals with bounds on \( \arg \left[ f'(z) \right], |f'(z)| \) and \( |a_n| \). Upper bounds are also determined for \( |a_{n+1}| - |a_n| \) and \( |a_3 - \mu a_2^2| \), \( \mu \) any complex number.

Let \( C \) denote the class of functions \( f(z) \) such that \( zf'(z) \) is a spiral function. In chapter four the radii of convexity and close-to-convexity of \( H \) is \( 2 - \sqrt{3} \). Also an example of a univalent subclass of \( H \) is given by appealing to the Schwarzian derivative.