Pure jump Lévy processes have been the focus of recent work in mathematical finance. They allow for more realistic representation of stock price dynamics and numerically tractable models for option pricing. In this dissertation we propose a new option pricing model using the generalized tempered stable process and investigate numerical methods in pricing vanilla and exotic instruments.

The Markov property of the price allows us to express option prices as solutions of partial integro-differential equations which involve, in addition to a second order differential operator, a non-local integral term. We develop an efficient implicit-explicit finite difference method based on the second order backward differentiation formula for solving them. Numerical results show that the scheme is consistent, convergent and stable.

Monte Carlo simulation is also investigated for the valuation of path dependent options. A compound Poisson approximation is applied in the simulation of stock sample paths. We show how small jumps can be further approximated by a Brownian motion. In order to reduce the standard errors in simulation, we make use of the control variate technique.

Finally, we calibrate the GTSP model to the S&P500 call options and present some properties of the risk neutral processes implied from option prices. The implied volatility surface is reproduced to demonstrate that the model is able to capture important features of the option market.