This thesis has been directed toward the goal of identifying objects in a finite-depth, constant or variable index of refraction ocean. The mathematical procedure is based on the physical idea of scattering one or more "plane waves" off the unidentified object and then trying to identify the object from its far-field pattern. In this thesis, we have developed a mathematical theory, numerical algorithm and some examples of computer implementation for a class of direct and inverse scattering problems for time harmonic acoustic waves in shallow oceans.

We consider the finite depth ocean to occupy a slab $R_3: = \{({\bf x},z) \in R^2, 0 \le z \le h\}$. The index of refraction may be considered constant or a function of the depth variable for purpose of exposition.

In order to solve the inverse problem we must first solve the direct (propagation) problem i.e., for given incoming plane waves $u^i$, to determine the scattered wave $u^s$. The starting field is set up using an integral equation method with a density; this in turn is "matched" to the propagating field using a weighted, least-squares method. The far-field for the finite depth ocean corresponds to only finite number of propagating modes. The far-field for the scattered acoustic pressure $u^s({\bf x},z)$ is given by an integral over the object's boundary $\partial \Omega$.

The underwater acoustic inverse problem is doubly ill-posed. First, as only a finite number of modes propagate, there is incomplete information in the far-field. Furthermore, even in space $R^3$ the inverse problem is ill-posed. We have made this problem well-posed by finding a suitable regularization; namely, we seek solutions in a suitable class of objects. Our inverse problem is reformulated as an optimization problem. We seek a Herglotz kernel $g(\{bf x\},z)$ and an "object" represented in spherical polar coordinates $\rho(\theta,\phi)$ which minimizes the functional.

Based on the theory developed as above, we have reconstructed objects in finite depth oceans, with a constant refraction index. We have also constructed the propagating modes for the variable index case. Some other problems in the shallow ocean such as the reconstruction of refraction index and optimal control problem are also considered in the thesis.