Replaceable Partial Spreads and the Construction of Non-Desarguesian Translation Planes
Richard Allen Weida
1987

This dissertation is concerned with the construction and classification problem for finite translation planes. As every two-dimensional translation plane can be constructed from a spread of PG(3,q), it seems natural to concentrate on the geometric properties of such spreads. New spreads can be constructed from a regular spread by replacing a subset of lines U of the regular spread with some other partial spread V covering the same points of (SIGMA) = PG(3,q). With this in mind, we have generalized Bruen's notion of a chain of reguli to that of a nest of reguli.

We define a nest of reguli to be a collection P of reguli in a regular spread S of PG(3,q) such that every line of S is contained in exactly 0 or 2 reguli of P. The size of the nest P is the number of reguli in it. We concentrate on nests of size at most q. Let U denote the lines of S which are contained in the reguli of P. While it is not known whether the set U is always replaceable, we show that when it is replaceable, the replacement set V consists of opposite half-reguli, one from each regulus in the nest. The spread obtained from S by replacing U with V is not subregular, at least for q (GREATERTHEQ) 7. Moreover, the full collineation group of the associated translation plane will be its inherited group, for q > 7.

We also construct numerous examples of nests of reguli, all of which are replaceable. Furthermore, by defining various operations on chains of reguli, we develop a construction scheme that is rich enough to generate all the regulus-containing spreads of PG(3,5) and 58 projectively inequivalent regulus-containing spreads of PG(3,7). In addition, several regulus-free spreads are obtained by our methods.