Radial basis function (RBF) approximations have been successfully used to solve partial differential equations in multidimensional complex domains. RBF methods are often called meshfree numerical schemes since they can be implemented without an underlying mesh or grid. We are particularly interested in the class of RBFs that allow exponential convergence for smooth problems. In the presence of rounding errors, stable and highly accurate approximations are often difficult even for simple geometries. In this thesis we explore this difficulty and possible remedies at theoretical and practical levels.

Theoretically, we explore a connection between Gaussian RBFs and polynomials. Using standard tools of potential theory, we find that these radial functions are susceptible to the Runge phenomenon, not only in the limit of increasingly flat functions, but also in the finite shape parameter case. We show that there exist interpolation node distributions that prevent such phenomena and allow stable approximations. This is believed to be the first proof that the stability of RBF interpolation depends on the location of the nodes. Using polynomials also provides an explicit interpolation formula that avoids the difficulties of inverting interpolation matrices, without imposing restrictions on the shape parameter or number of points.

We also show that node location plays an important role in the stability of RBF-based methods for time-dependent problems. Differentiation matrices obtained with infinitely smooth radial basis function (RBF) collocation methods have, under many conditions, eigenvalues with positive real part, preventing the use of such methods for time-dependent problems. We prove that differentiation matrices for conditionally positive definite RBFs are stable for periodic domains. We also show that for Gaussian RBFs, special node distributions can achieve stability in 1-D and tensor-product non-periodic domains.

As a more practical approach, we investigate RBF least-squares approximations. We consider differentiation matrices based on least-squares discretizations and show that such schemes can lead to stable methods on less regular nodes. By separating centers and nodes, least-squares techniques open the possibility of the separation of accuracy and stability characteristics. An Arnoldi-like iterative algorithm is also presented to generate an orthogonal basis for the subspace spanned by Gaussian RBFs. Algorithms for eigenvalue problems are also investigated.