Homogenizing acoustic properties of cancellous bone
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The two self-contained parts of this thesis each investigate a challenging phenomenon using scientific computing. In the powerful technique of scientific computing, mathematical models are first developed, and then analyzed and solved through the construction of numerical algorithms. In Part I, we study the stability of Gaussian elimination with partial pivoting via experiments on random matrices. In Part II, we investigate the dynamics of the human tear film subject to reflex tearing, and the tear flow on the eye-shaped geometry subject to different boundary conditions.

The growth factor of an $n \times n$ matrix quantifies the amount of potential error growth when a linear system is solved by Gaussian elimination with partial pivoting. It has long been known that there exists matrices with growth factors up to $2n-1$, and therefore, Gaussian elimination with partial pivoting is theoretically unstable. However, years of experience and analysis suggest that these matrices with exponentially large growth factors are exceedingly rare. In Part I, we build upon the work of Trefethen and Bau [84] to explore the tails of growth factor probability distributions with a multicanonical Monte Carlo method. Our results suggest that the occurrence of an $8 \times 8$ matrix with a growth factor of 40 is on the order of a once-in-the-age-of-the-universe event.

The human tear film is a complicated multilayer thin film on the surface of the eye that plays an essential role in the health and protection of the eye. In Part II, we focus on modeling the relaxation of the tear film after a blink (i.e., the period when the lids remain open) and explore the effects of viscosity, surface tension, gravity, evaporation, and different boundary conditions. The governing nonlinear partial differential equation derived using an lubrication theory is numerically solved on an overset grid by an implementation of the method of lines with a finite difference discretization in space and an adaptive backward difference formula solver in time. The choice to use an overset grid is motivated by our desire to simulate the tear film dynamics on a blinking domain. The OVERTURE framework is used in two dimensions, because it has many functions needed for interpolation, data handling, and so forth, required for this complex domain.

Chapter 5 investigates the phenomenon of reflex tearing along a single line down the center of the cornea. The computations of our tear film model show qualitative agreement with in vivo tear film thickness measurements illustrating the effects of reflex tearing. Furthermore, the role of the black lines in the presence of tear supply from the lid margins was found to be more subtle than as a barrier to tear fluid flow between the anterior of the eye and the meniscus at the lid margin. In Chapters 6 and 7, we focus our efforts on simulating the tear flow on an eye-shaped domain subject to different pressure and flux boundary conditions. The computed flows show sensitivity to both our choice of boundary condition and to the presence of gravity. Moreover, the simulations recover features seen in one-dimensional simulations and capture some experimental observations including hydraulic connectivity between the menisci by the upper and lower lids.