An Inverse Problem Arising in Corrosion Detection
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In this work, we apply the principles of electrical impedance tomography to the nondestructive evaluation of corrosion. The corrosion is assumed to occur on an inaccessible part of a specimen, and measurements are taken on the accessible part of the boundary. By modeling corrosion damage as material loss, the problem amounts to determination of an unknown surface of a body from electrostatic measurements on a portion of the boundary. This study is motivated by the need to develop new technologies for nondestructively measuring corrosion damage in aircraft.

We derive the mathematical model for our problem from Maxwell's equations for electromagnetic waves under the assumption of an isotropic, linear medium. In the analysis of this work, we shall study the specific model problems of two-dimensional and three-dimensional constant conductivity thin plates with damage constrained to the top surface and electrostatic measurements taken on the opposite side. The mathematical model for zero frequency currents reduces to a Neumann boundary value problem for Laplace's equation.

The forward problem of determining the potential function resulting from an applied current to a plate with known damage is solved for both the 2-D and 3-D problems using boundary element methods. These solutions motivate the derived inversion algorithms and provide synthetic data for testing the developed schemes.

For both model problems, we begin by linearizing the relationship between voltage differentials and damage profiles for small corrosion damage (5-10% of plate thickness). In the 2-D case, we first use this linearized relationship to determine the optimal current pattern to maximize data content for a specified damage. We then proceed to derive and analyze multiple inversion schemes for the two cases of 'full' continuous data and of a finite set of point voltage data. The stability of these numerical schemes is analyzed, and numerical examples are provided to indicate the level of reconstruction attainable by each scheme.

Based on an observed proportional relationship between the damage shape and the derivative of the voltage difference, we derive a three-dimensional inversion scheme. In this case, an assumption of 'small' plate thickness reduces the linearized relationship to a two-dimensional hyperbolic equation in the damage function. With a specific choice of input current, the inversion becomes a direct marching algorithm. Improvements are made to the reconstruction scheme by introducing a regularization strategy which leads to the final inversion algorithm for the 3-D plate problem.

We conclude that small corrosion damage on a thin plate can be estimated accurately in terms of location and depth. Future work will be directed toward improving resolution of the inversion scheme, analyzing the physical applicability of the technique, and incorporating more general physical assumptions into the mathematical model.