Subsets of Finite Groups Exhibiting Additive Regularity
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2008

We explore a combinatorial notion called additive regularity. Chapter 1 provides an historical context for the study of additively regular sets and introduces the idea of the so-called special subsets. The following five chapters comprise the main body of the text and split naturally into two parts.

Chapters 2, 3, and 4 form a fundamental theory of sum sets. In Chapter 2 we introduce the basic notions of sum sets and partial sum sets, and we prove several nonexistence results for sum sets. We describe how the normal subgroup structure of a group affects the group's potential to admit sum sets. We also show that sum sets are almost entirely a nonabelian phenomenon, as any abelian sum set is necessarily a reversible difference set whose additive regularity is trivial.

Existence results and explicit constructions for sum sets are presented in Chapter 3. Generic constructions and lifting techniques are developed and applied to certain classes of nonabelian groups. In particular, we construct infinite families of sum sets in dihedral groups and a class of Frobenius groups. In the fourth chapter we list all sum sets in groups of order at most thirty; for those not constructable by the results of Chapter 3 we provide brief descriptions.

The second part of this work is the investigation of the additive properties of skew Hadamard difference sets in Chapters 5 and 6. In Chapter 5 we prove skew Hadamard difference sets are partial sum sets and explore several consequences. Most notably, this result allows a complete characterization of the full multiplier group of abelian skew Hadamard difference sets. We prove in Chapter 6 that skew Hadamard difference sets are characterized by the behavior of their special subsets, and we describe how these subsets' properties may be used to construct several combinatorial structures. The final chapter contains brief discussions of some open questions.