A singularly perturbed Cauchy problem for an evolution equation in a Hilbert space
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We consider a second-order semilinear evolution equation in a real Hilbert space, with Cauchy data and a small parameter (epsilon):

\[(\varepsilon)^2u''(t)+u'(t)+Au(t) = F(u(t)) \quad (E)\]
\[u(0) = u(,0), \ (\varepsilon)u'(0) = u(,1). \quad (I)\]

Many nonlinear damped vibration problems of mechanics can be formulated in the abstract setting of (E) (I). This problem is singularly perturbed since there is a reduction of order when (epsilon) = 0 and the reduced equation

\[U'(t)+AU(T) = F(U(t))\]

together with the single condition \(U(0) = u(,0)\), forms a well-posed problem. This behavior suggests the existence of an initial layer. Our main result is the proof of the uniform validity of an N-term asymptotic expansion of \(u\) with initial layer corrections on an arbitrary time interval \([0,T]\). We thereby succeed in extending to an abstract setting recent results of Hsiao and Weinacht dealing with the semilinear partial differential equation

\[(\varepsilon)^2u(,tt)+u(,t)-u(,xx) = f(u)\]

This abstract analysis is subsequently used to obtain estimates for a continuous-time numerical scheme for a class of singularly perturbed second-order hyperbolic pde's in \((/R)((n)). Convergence of the scheme by finite element methods is obtained.