This thesis deals with an interesting application of mathematics, tear film dynamics. Our goal is to understand and simulate the various observed tear film behaviors, such as film cooling, black line formation and lipid patterns etc. During this process, we build models to describe the fluid dynamics for several blink cycles, experiment on the choices of parameters, compare the simulation with in vivo observation, and develop innovative numerical algorithms.

The human tear film is an intricate multilayer thin film on the surface of the cornea that plays an important role in maintaining proper eye function and keeping eye health. To successfully model tear film dynamics requires consideration of viscosity, surface tension, evaporation, Marangoni effect, eyelid motion and different boundary conditions. Because of the tear film nature, lubrication theory is applied to the tear film study to formulate a nonlinear partial differential equation governing film dynamics. Depending on the focus of the study, different equations such as heat conduction, lipid transport, have been coupled with the film equation. Further, different types of lid motions are considered in the model. As a result, in each study we deal with a fourth-order nonlinear system on a moving domain; the accuracy and stability requirement poses a computationally challenging problem. The technique we use pertains to spectral/pseudospectral and meshless method. In particular, the latter is promising when the shape of domain is irregular, such as an eye-shaped domain.

This thesis is structured as follows. Introduction will be presented in Chapter 1. In Chapter 2, we will develop a model to study the thermal dynamics of the tear film; we compare our simulation with the cooling phenomena observed in vivo. We will use this model to explore the tear film dynamics over multiple blink cycles. In Chapter 3, we study the dynamics for incomplete blink and compare with in vivo measurements. In Chapter 4 we develop a 2D model by coupling the lipid layer with the underneath tear film. We study the tear film dynamics subject to the Marangoni effect exerted by the lipid layer, and compare our simulation with observed lipid patterns. Finally in Chapter 5, we explore a moving eye-shaped domain with the following two approaches: preliminary effort with conformal mapping is presented, and a tree based algorithm is developed to improve the efficiency of preconditioned GMRES iteration. Both could be applied in modeling the irregular eye-shaped domain.