2.5-dimensional electrical impedance tomography
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This work details the application of a specific technique for solving the inverse problem of Electrical Impedance Tomography (EIT) that is known as the 2.5-dimensional problem. In this scenario the conductivity is assumed to be invariant with respect to a dimension which goes perpendicular to the borehole plane. While the 2-dimensional assumption is often made, in this situation the electrical field and current sources are allowed to be fully 3-dimensional. Such a problem has been addressed, but typically in such a way that requires the solution of several 2-dimensional problems. In this work, we introduce a novel method of solution of the 2.5-dimensional problem for which the computational cost rivals that of 2-dimensional problems, while the extra freedom given to the current source and potential functions can increase the accuracy of the model.

In this work, the forward problem is approached by first performing a Fourier transform with respect to the invariant axis which collapses the 3-dimensional problem into a series of two-dimensional problems, one for each wave-number. The two-dimensional problem is solved using finite difference or finite element techniques. A key procedure by which the approximate solution is obtained efficiently is the Lanczos algorithm as first proposed by Druskin & Knizhnerman. Finally, the full 3-dimensional field is obtained by analytically inverting the Fourier transform. What is contained in this work is: (1) a detailed mathematical analysis of the forward problem, (2) a detailed description of the method to obtain numerical solutions and (3) numerical analysis which supply the missing 'gaps' in the current literature.

To solve the inverse problem, we use Newton’s method. The number of available data and the number of unknowns, in addition to the illposedness of the problem forces us to consider a pseudo-inverse approach to invert the Jacobian matrix. We also use the Jacobian to carefully study the 'information content' of the experiment. An algorithm for solving the inverse problem, complete with an efficient method for computing the Jacobian, is outlined in this work.

We conclude that for a simple configuration, the 2.5-dimensional problem can be solved with 'less-than-spectacular' results. The sensitivity analysis performed in this work shows that we can not resolve much of the conductivity and in fact a linearized method will probably perform with comparable results. Future research could be directed toward finding better configurations for data collection, and possibly combining the results from EIT with other information such as those alluded to earlier in this chapter.