Two methods are given to solve the Helmholtz equation in a three-dimensional region, exterior to a smooth closed and bounded surface on which Neumann boundary conditions are imposed. Both methods depend on a new integral equation formulation of the problem. In one case, the integral equation, which is a regularized Fredholm equation of the second kind, is solved by direct iteration. The convergence of this sequence of iterates is proven for strictly convex surfaces for sufficiently small values of k. This method is a direct generalization to the Helmholtz equation of Neumann's method for solving potential problems which appears as a special case, zero frequency, of the present method. In the second method, the solution is found as an infinite series which converges for smooth but otherwise arbitrary surfaces and for small values of the wave number. The generalized Neumann method is demonstrated to be considerably simpler to use compared with either the second method or the construction of the Fredholm resolvent. This is shown in a number of potential and scattering problems involving spherical and spheroidal surfaces. In every case, it is verified that the method produces the correct solution.